Advanced Plasma Shape Control to Enable High-Performance Divertor Operation on NSTX-U

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Abstract

This work presents the development of an advanced framework for control of the global plasma shape and its application to a variety of shape control challenges on NSTX–U. Operations in high-performance plasma scenarios will require highly-accurate and robust control of the plasma poloidal shape to accomplish such tasks as obtaining the strong-shaping required for the avoidance of MHD instabilities and mitigating heat flux through regulation of the divertor magnetic geometry. The new control system employs a high-fidelity model of the toroidal current dynamics in NSTX–U poloidal field coils and conducting structures as well as a first-principles driven calculation of the axisymmetric plasma response. The model-based nature of the control system enables real-time optimization of controller parameters in response to time-varying plasma conditions and control objectives. The new control scheme is shown to enable stable and on-demand plasma operations in complicated magnetic geometries such as the snowflake divertor. A recently-developed code that simulates the nonlinear evolution of the plasma equilibrium is used to demonstrate the capabilities of the designed shape controllers. Plans for future real-time implementations on NSTX–U and elsewhere are also presented.
Introduction

Goals of this work

• The use of advanced divertor configurations, such as the snowflake divertor (SFD), is being considered as a possible means of reducing peak heat flux onto divertor surfaces in NSTX-U.

• Develop an algorithm that is capable of real-time control of all divertor configurations of interest in NSTX-U.

• Address primary limitations of the algorithm as previously implemented on DIII-D:
  – Stable control of the SFD-Plus configuration.  
  – Recovery from high-field-side to low-field-side SFD-Minus.

• Transition to model-based (non-PID) plasma shape control for NSTX-U.

Highlights

• New modeling of X-point position response to PF coil currents.

• PID-based control of the SFD with closed-loop controller tuning using relay feedback.

• Initial development of model-based Linear-Quadratic-Integral control of the plasma shape.
Formalism for plasma shape control modeling

Coupled circuit equations describing dynamics of toroidal currents in coils, passive structures, and plasma.

\[
\begin{align*}
M_{cc} \dot{I}_c + M_{cv} \dot{I}_v + \dot{\Psi}_{cp} + R_c I_c &= V_c \\
M_{vc} \dot{I}_c + M_{vv} \dot{I}_v + \dot{\Psi}_{vp} + R_v I_v &= 0 \\
M_{pc} \dot{I}_c + M_{pv} \dot{I}_v + \dot{\Psi}_{pp} + R_p I_p &= V_{no}
\end{align*}
\]

- Vacuum mutual inductances
- Time-derivatives of plasma flux at coils, vessel, and plasma
- Conductor resistances
- Applied voltages

coils
vessel
plasma
Linearized circuit equations

\[
\begin{align*}
M_{cc} \dot{I}_c + M_{cv} \dot{I}_v + \frac{\partial \Psi_{cp}}{\partial I_c} \dot{I}_c + \frac{\partial \Psi_{cp}}{\partial I_v} \dot{I}_v + \frac{\partial \Psi_{cp}}{\partial I_p} \dot{I}_p + R_c I_c &= V_c \\
M_{vc} \dot{I}_c + M_{vv} \dot{I}_v + \frac{\partial \Psi_{vp}}{\partial I_c} \dot{I}_c + \frac{\partial \Psi_{vp}}{\partial I_v} \dot{I}_v + \frac{\partial \Psi_{vp}}{\partial I_p} \dot{I}_p + R_v I_v &= 0 \\
M_{pc} \dot{I}_c + M_{pv} \dot{I}_v + \frac{\partial \Psi_{pp}}{\partial I_c} \dot{I}_c + \frac{\partial \Psi_{pp}}{\partial I_v} \dot{I}_v + \frac{\partial \Psi_{pp}}{\partial I_p} \dot{I}_p + R_p I_p &= V_{n.o.}
\end{align*}
\]

Linearized response of plasma flux due to changes in external currents and bulk plasma current.

\[
X_{jk} = \frac{\partial \Psi_{jp}}{\partial \xi_r} \frac{\partial \xi_r}{\partial I_k} + \frac{\partial \Psi_{jp}}{\partial \xi_z} \frac{\partial \xi_z}{\partial I_k}, \quad j, k \in \{c, v, p\}
\]

Outputs of a linear plasma response model
State-space representation of the dynamics

Express the linearized circuit equations in state-space form for use with model-based control design tools

\[
\delta \dot{x} = A(t) \delta x + B(t) \delta u
\]

Perturbed currents
\[
\delta x = \begin{bmatrix}
I_c - I_{c_{eq}} \\
I_v - I_{v_{eq}} \\
I_p - I_{p_{eq}}
\end{bmatrix}
\]

Perturbed voltages
\[
\delta v = \begin{bmatrix}
V_c - V_{c_{eq}} \\
0 \\
0
\end{bmatrix}
\]

Time-dependent mutual inductance matrix
\[
\widehat{M}(t) = M + X(t)
\]

Resistance matrix
\[
R
\]

Map from voltages to coils
\[
V
\]

Vacuum mutual inductances

Effective mutual inductance due to plasma motion

\[
A(t) = - \left[ \widehat{M}(t) \right]^{-1} R
\]

\[
B(t) = \left[ \widehat{M}(t) \right]^{-1} V
\]
Validation of the no-plasma model

Wall model validated by comparing synthetic and measured magnetic diagnostic signals for NSTX-U vacuum shots.

- Predicted
- Measured
Dominant eigenmodes of the vacuum vessel

Future validation efforts will seek to identify source of the asymmetry in the vessel model.
Output equation for ISOFLUX control

State-space dynamics equation paired with output equation relating the inputs (voltages) and states (currents) to quantities of interest for control.

\[ \delta y = C(t) \delta x + D(t) \delta u \]

\[
\begin{bmatrix}
\Delta \psi_1 \\
\Delta \psi_2 \\
\Delta \psi_3 \\
\Delta r_{xU} \\
\Delta z_{xU} \\
\Delta r_{xL} \\
\Delta z_{xL}
\end{bmatrix} \quad \text{Control point fluxes}
\]

\[
\begin{bmatrix}
\partial_1 \psi_1 \\
\partial_1 \psi_2 \\
\partial_1 \psi_3 \\
\partial_1 r_{xU} \\
\partial_1 z_{xU} \\
\partial_1 r_{xL} \\
\partial_1 z_{xL}
\end{bmatrix} \quad \text{X-point positions}
\]

In general, matrix entries are time-dependent.
Modeling of X-point response

\[ \frac{\partial r_x}{\partial I} = \frac{\partial r_x}{\partial B} \frac{\partial B}{\partial I} = \frac{\partial r_x}{\partial B} \left( \frac{\partial B}{\partial I} \right)_{\text{vac}} + \frac{\partial B}{\partial \xi_r} \frac{\partial \xi_r}{\partial I} + \frac{\partial B}{\partial \xi_z} \frac{\partial \xi_z}{\partial I} \]

\[ \frac{\partial z_x}{\partial I} = \frac{\partial z_x}{\partial B} \frac{\partial B}{\partial I} = \frac{\partial z_x}{\partial B} \left( \frac{\partial B}{\partial I} \right)_{\text{vac}} + \frac{\partial B}{\partial \xi_r} \frac{\partial \xi_r}{\partial I} + \frac{\partial B}{\partial \xi_z} \frac{\partial \xi_z}{\partial I} \]

Green’s functions for the coil-only vacuum fields

Outputs of a linear plasma response model

\[ \frac{\partial B}{\partial r} r_x \quad \text{and} \quad \frac{\partial B}{\partial z} z_x \quad \text{Computed analytically using saddle-point expansion of the magnetic flux} \]

\[ x = r - r_0 \quad v = z - z_0 \]

\[ (r_0 + x) \frac{\partial}{\partial x} \left( \frac{1}{r_0 + x} \frac{\partial \psi}{\partial x} \right) + \frac{\partial^2 \psi}{\partial v^2} = 0 \]

\[ B_r = -\frac{1}{r_0 + x} \frac{\partial \psi}{\partial v} \quad B_z = \frac{1}{r_0 + x} \frac{\partial \psi}{\partial x} \]

Flux

Field
Response of X-point position to B-field

1. Expand the flux function as a series to second-order. Solve for the series coefficients using measurements of $B_r$ and $B_z$ at two points.

$$\psi(x, v) = l_1 x + l_2 v + q_1 x^2 + 2q_2 xv + q_3 v^2$$

$$B_r = -\frac{1}{r_0 + x} (l_2 + 2q_2 x + 2q_3 v)$$

$$B_z = \frac{1}{r_0 + x} (2(r_0 + x)q_1 + 2q_2 v + 2q_3 r_0)$$

2. Solve for the $(r,z)$ coordinates of the X-point.

$$r = r_0 + \frac{l_2 q_2 - l_1 q_3}{2(q_1 q_3 - q_2^2)}$$
$$z = z_0 + \frac{l_2 q_1 - l_1 q_2}{2(q_2^2 - q_1 q_3)}$$

3. Compute derivatives of $(r_x, z_x)$ with respect to $B_r$ and $B_z$.

$$\frac{\partial B}{\partial r} r_x$$
$$\frac{\partial B}{\partial z} z_x$$
Snowflake Shape Descriptors

Four parameters used for control

\( r_{\text{Snow}} \)

\( z_{\text{Snow}} \)

\( \rho \)

\( \theta \)
SFD control system with PID control

SFD position request

\((C^T C)^{-1} C^T\)

Pseudoinverse from SFD errors to PF coil current errors

\(\Delta I_{PF2L} \rightarrow \text{PID} \rightarrow \Delta V_{PF2L} \rightarrow V_{PF2L} \rightarrow \text{PID} \rightarrow \Delta V_{PF1cL} \rightarrow V_{PF1cL} \rightarrow \text{PID} \rightarrow \Delta V_{PF1aL} \rightarrow V_{PF1aL} \rightarrow V_{PF}\)

Grad-Shafranov equilibrium reconstruction

\(J. R. Ferron et al. Nucl. Fusion. (1998)\).

Locate snowflake using local expansion of flux


Magnetics

NSTX-U

Magnetics

SFD Observer

\(\psi (r, z)\)

\(\{r_c, z_c, \rho, \theta\}\)
PID Control and Controller Tuning

Compute control action from the error between desired and measured signal

\[ u(t) = K_P \left( e(t) + \frac{1}{T_I} \int_0^t e(\tau) \, d\tau + T_D \frac{d}{dt} e(t) \right) \]

Terms proportional to error, integral of error, and derivative of error

**Closed-Loop** tuning of PID gains using Relay Feedback

Replace PID controller with relay

\[ K_u = \frac{4h}{\pi a} \]

Ultimate Gain
**Scenario 1:** Scan of the X-point separation in the SFD-Minus configuration.

- $t = 300ms$
- $t = 380ms$
- $t = 425ms$
- $t = 500ms$
- $t = 600ms$
- $t = 700ms$
**Scenario 2:** Scan of the X-point separation in the SFD-Plus configuration.

![Image showing the snowflake radius scan in SFD-Plus with different time stamps: t = 200ms, t = 275ms, t = 350ms, t = 450ms, t = 500ms, t = 600ms.](image-url)
Snowflake angle scan at constant separation

**Scenario 3:** Scan of the angular orientation with constant X-point separation.

\[ t = 200\text{ms} \]

\[ t = 300\text{ms} \]

\[ t = 400\text{ms} \]

\[ t = 500\text{ms} \]

\[ t = 600\text{ms} \]

\[ t = 700\text{ms} \]
Future work

- Test the designed controllers for the SFD in-the-loop with free-boundary Grad-Shafranov equilibrium solver (for verification of controller performance).

- Integration of the SFD control into a model-based shape controller designed with LQI for reference tracking of plasma shape parameters.

- Implementation of PID-based and LQI-based controllers for the SFD in the DIII-D and NSTX-U plasma control systems.

- Test the new algorithms in DIII-D SFD scenarios.