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PLASMA PHYSICS
LABORATORY

Control of Advanced Divertors in NSTX-U and ITER

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PRINCETON UNIVERSITY

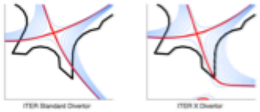
NSTX-U

Motivation

- Reduce heat flux to the divertors in NSTX-U and ITER

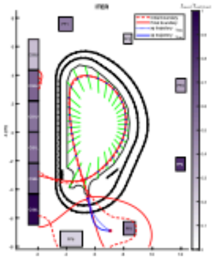
snowflake — secondary x-point placed next to primary x-point.
x-divertor — secondary x-point placed near outer strike point.

- Snowflake is possible on NSTX-U [1], but coil currents too high for ITER [2].
- X-divertor is possible on ITER [3], but no studies on how to achieve transition.

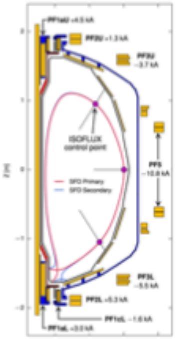


Method

- Maintain shape via isoflux method (minimizes flux error between plasma boundary and control points).
- Use reference tracking on the x-point positions.
- NSTXU – Linear Quadratic Integral control (LQI)
- ITER – Model Predictive Control (MPC)



ITER Control Setup



NSTXU Control Setup

Dynamics Model

The vacuum vessel elements, poloidal field (PF) coils, and plasma are treated as toroidally looped circuits:

$$v_s = R_s I_s + \dot{\Psi}_{ss,\ coil} + \dot{\Psi}_{ss,\ plasma}$$

$$\Uparrow \qquad\qquad \Uparrow$$

$$\dot{\Psi}_{ss,\ coil} = M_{ss}\dot{I}_s$$

Flux change due to induced currents.

$$\dot{\Psi}_{ss,plasma} \approx \left.\frac{\partial \Psi_s}{\partial I_s}\right|_{eq} \delta \dot{I}_s := X_{ss}\delta \dot{I}_s$$

Flux change due to plasma motion. Computed via linearization to Grad-Shafranov equation [3]

Matrix form of circuit equation gives state-space dynamics:

$$\begin{bmatrix}\delta v_s \\ 0\end{bmatrix} = \begin{bmatrix}R_s & 0 \\ 0 & R_p\end{bmatrix}\begin{bmatrix}\delta I_s \\ \delta I_p\end{bmatrix} + \begin{bmatrix}M_{ss}+X_{ss} & M_{sp}+X_{sp} \\ M_{ps}+X_{ps} & M_{pp}+X_{pp}\end{bmatrix}\begin{bmatrix}\delta \dot{I}_s \\ \delta \dot{I}_p\end{bmatrix}$$

$$\Updownarrow$$

$$\delta \dot{I} = A(t)\delta I + B(t)\delta v$$

Output Model

Control plasma current, x-point positions, strike point positions, and shape.

$$Z = [I_p \quad r_x \quad z_x \quad r_{strike} \quad z_{strike} \quad \psi_{bry} \quad \psi_{cp\times 31}]^T$$

Write the output equation in the linearized frame (to match dynamics) .

$$e = Z - Z_{target}$$

$$\delta e = \frac{\partial (Z - Z_{target})}{\partial I}\delta I \quad \Leftrightarrow \quad y = C(t)\delta I$$

Flux response $\frac{\partial \psi}{\partial I}$ obtained via linearization to Grad-Shafranov equation [4].

NSTX-U Control

Control strategy: use linear quadratic integral (LQI) with set-point tracking on the x-point positions. Use proportional control to maintain plasma shape.

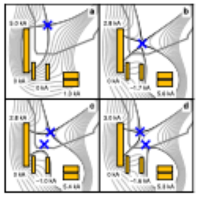
Set point tracking:

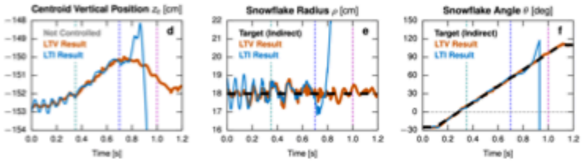
$$\begin{aligned} Ax^* + Bu^* &= 0 \\ Cx^* &= r \end{aligned} \implies \begin{bmatrix}x^* \\ u^*\end{bmatrix} = \begin{bmatrix}A & B \\ C & 0\end{bmatrix}^{-1}\begin{bmatrix}0 \\ I\end{bmatrix} r := \begin{bmatrix}F_x r \\ F_u r\end{bmatrix}$$

$$u = -Kx \implies u = -K(x - x^*) + u^*$$

Final feedback law:

$$u = -K_p(x - F_x r) + F_u r + K_I \int_0^t (y - r)d\tau$$





ITER Control

Control strategy: coil current limits and output constraints are a concern. Use Model Predictive Control (MPC) with target tracking.

Reference trajectory cost function:

$$J_k = \sum_{i=1}^{N}\left[(y_{k+i} - r_{k+i})^T Q_i (y_{k+i} - r_{k+i}) + u_{k+i-1}^T R_i u_{k+i-1}\right]$$

Use model dynamics and output equation to predict N steps into the future.

$$\underbrace{\begin{bmatrix}y_{k+1} \\ y_{k+2} \\ \vdots \\ y_{k+N}\end{bmatrix}}_{\hat{Y}} = \underbrace{\begin{bmatrix}CA \\ CA^2 \\ \vdots \\ CA^N\end{bmatrix}}_{E} x_k + \underbrace{\begin{bmatrix}CB & & & \\ CAB & CB & & \\ \vdots & \vdots & \ddots & \\ CA^{N-1}B & CA^{N-2}B & \dots & CB\end{bmatrix}}_{F}\underbrace{\begin{bmatrix}u_k \\ u_{k+1} \\ \vdots \\ u_{k+N-1}\end{bmatrix}}_{\hat{U}}$$

Cost function minimization simplifies to:

$$\text{minimize} \quad J_k = \hat{U}^T H \hat{U} + 2f^T \hat{U}$$

$$H := F^T \hat{Q} F + \hat{R}, \quad f := F^T \hat{Q}(Ex_k - \hat{r})$$

ITER MPC Constraints

- Constrained variables include the coil currents, applied coil voltages, power characteristics, and shape.
- Constraints written in the linearized reference frame, in terms of the optimization variable $\hat{U}$

Activated Constraints

PF Coils			Outputs
Coil #	I	V	--
PF1	< 48 kA	< 1.5 kV	$\dot{P}$ < 200 MW/s
PF2	< 55 kA	< 1.5 kV	P < 250 MW
PF3	< 55 kA	< 1.5 kV	δI_p < 3%
PF4	< 55 kA	< 1.5 kV	r_{strike} on plate
PF5	< 52 kA	< 1.5 kV	z_{strike} on plate
PF6	< 52 kA	< 1.5 kV	cp_{33} gap
CS1U	< 45 kA	< 1.5 kV	cp_{34} gap
CS1L	< 45 kA	< 1.5 kV	cp_{35} gap
CS2U	< 45 kA	< 1.5 kV	cp_{36} gap
CS2L	< 45 kA	< 1.5 kV	
CS3U	< 45 kA	< 1.5 kV	
CS3L	< 45 kA	< 1.5 kV	

- Example:

$$Gv \leq g \qquad v = v_0 + u$$

$$\Updownarrow$$

$$\hat{G}\hat{U} \leq \hat{g} - \hat{G}\hat{v}_0 \qquad \hat{G} := \text{blockdiag}\underbrace{(G \dots G)}_{\times N} \qquad \hat{g} := \underbrace{[g^T \dots g^T]}_{\times N}{}^T \qquad \hat{v}_0 := \underbrace{[v_0^T \dots v_0^T]}_{\times N}{}^T$$

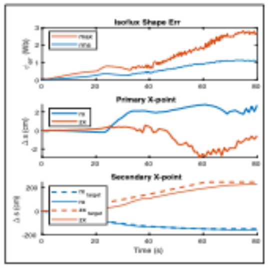
Results

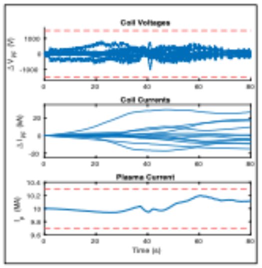
NSTX-U

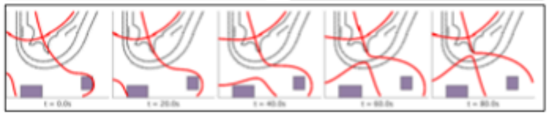
- A time-variant plasma model coupled with LQI control can create the snowflake divertor and gives good tracking error (<1cm).

ITER

- For a 10MA plasma, can transition to x-divertor while maintaining constraints.
- All physical constraints have been implemented except for series connection between CS1U and CS1L.







ITER X-Divertor transition sequence.

Future Work

- Identify the time-variant plasma models present in the NSTXU ramp-up phase, so that shape control can be implemented during ramp-up (allows for more consistent entry into H-mode.)
- Implement series connection on ITER CS1U/L and identify under which conditions the x-divertor can still be created.

control.princeton.edu/presentations

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[1] P. Vail et al., "Design and simulation of the snowflake divertor control for NSTX-u," *Pla. Phys & Contr. Fusion*, Vol 61, Iss 3 (2019). [2] K. Lackner and H. Zohm, "Calculation of realistic snowflake equilibria for next-step devices," *FST*, Vol 63, Iss. 1 (2013). [3] B. Covele et al., "An exploration of advanced x-divertor scenarios on ITER," *N. Fusion*, Vol 54, Iss. 7 (2014).
[4] A.S. Welander et al., "Nonrigid, linear plasma response model based on perturbed equilibria for axisymmetric tokamak control design," *FST*, Vol 47, Iss. 3 (2005).