Control of Advanced Divertors – NSTXU, ITER, D3D

J.T. Wai, P.J. Vail, E. Kolemen (jwai@pppl.gov)

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Outline

• Motivation for advanced divertors
  • Heat flux spreading
• Dynamics model
• Output model
• **NSTX**: Linear Quadratic Integral (LQI) control of the snowflake divertor
• **ITER**: Model predictive control (MPC) of the X-divertor
• **DIII-D**: Improving snowflake reconstruction with IRTV diagnostic
• **NSTX**: Optimization of the cryopump location for snowflakes
Advanced magnetic field configurations can reduce power flux to the divertors

- Divertor heat load is a design challenge for high performance tokamaks
  - ITER ~ 10 MW/m^2

- Several ideas to reduce heat load
  - Minimize divertor plate angle (but > 1 deg)
  - Strike point sweeping
  - Advanced divertor configurations
    - Snowflake divertor
    - x divertor
    - super x divertor
The circuit equation applied to each conductor gives the state-space model dynamics

**Circuit Model**

\[ v_s = R_s I_s + \dot{\Psi}_{ss, \text{coil}} + \dot{\Psi}_{ss, \text{plasma}} \]

\[ \dot{\Psi}_{ss, \text{coil}} = M_{ss} \dot{I}_s \]

Flux change due to induced currents.

\[ \dot{\Psi}_{ss, \text{plasma}} \approx \left. \frac{\partial \Psi_s}{\partial I_s} \right|_{eq} \delta I_s := X_{ss} \delta I_s \]

Flux change due to plasma motion. Computed via TokSys and [1]

**State Space Form**

\[
\begin{bmatrix}
\delta v_s \\
0
\end{bmatrix} =
\begin{bmatrix}
R_s & 0 \\
0 & R_p
\end{bmatrix}
\begin{bmatrix}
\delta I_s \\
\delta I_p
\end{bmatrix} +
\begin{bmatrix}
M_{ss} + X_{ss} & M_{sp} + X_{sp} \\
M_{ps} + X_{ps} & M_{pp} + X_{pp}
\end{bmatrix}
\begin{bmatrix}
\delta I_s \\
\delta I_p
\end{bmatrix}
\]

\[ \dot{\delta I} = A(t) \delta I + B(t) \delta v \]

The linearized output equation is determined by a derivative expansion of the absolute error

- Controlled outputs

\[ Z = \begin{bmatrix} I_p & r_x & z_x & r_{\text{strike}} & z_{\text{strike}} & \psi_{\text{br}} & \psi_{\text{cp}} \times 31 \end{bmatrix}^T \]

- Write the output model in the linearized frame (matches dynamics).

\[ e = Z - Z_{\text{target}} \quad \delta e = \frac{\partial (Z - Z_{\text{target}})}{\partial I} \delta I \quad \Leftrightarrow \quad y = C(t) \delta I \]

- Reference trajectory defined by setting error to zero

\[ 0 = e := y + e_0 \quad \Leftrightarrow \quad r = -e_0 \]

- X-Point response

\[ \frac{\partial (r_x, z_x)}{\partial I} = \frac{\partial (r_x, z_x)}{\partial (\psi_r, \psi_z)} \frac{\partial (\psi_r, \psi_z)}{\partial I} = \left[ -\frac{\partial (\psi_r, \psi_z)}{\partial (r, z)} \right]^{-1} \begin{bmatrix} \frac{\partial r}{\partial I} \psi \\ \frac{\partial z}{\partial I} \psi \end{bmatrix} \]

Determined by [1]. Linearization to G-S.

Since there is a large separation of timescales, current & shape control can be designed separately from vertical stabilization

- Superconducting coil response time (s) vs. resistive wall decay time (ms)
- Simulation: negate eigval, exclude use of vs1/vs2 as actuators
- 3 control objectives
  - Minimize flux error between control pts and plasma boundary
  - Reference tracking of x-point positions
  - (ITER) Maintain Ip

![Diagram of tokamak control system]

\[ r(t) \rightarrow \text{Shape and Current Control} \rightarrow \text{Plasma in Tokamak} \rightarrow \text{Vertical Stabilization} \]
**Decoupled control scheme:**
- Linear quadratic integral (LQI) for divertor variables
- Proportional control on isoflux shape

Reference tracking:

\[
\begin{align*}
Ax^* + Bu^* &= 0 \\
Cx^* &= r
\end{align*}
\]

\[
\begin{bmatrix} x^* \\ u^* \end{bmatrix} = \begin{bmatrix} A & B \\ C & 0 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ I \end{bmatrix} r := \begin{bmatrix} F_x r \\ F_u r \end{bmatrix}
\]

Final feedback control law:

\[
u = -K_p (x - F_x r) + F_u r + K_I \int_0^t (y - r) d\tau
\]

Kp and KI from LQR of augmented system:

\[
\hat{A} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix}, \quad \hat{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}
\]
**NSTXU:** Robust snowflake divertor control requires the use of online model updates – P.J. Vail [1]

- Simulation shows high degree of control over snowflake configuration
- Highlights need for online model changes (LTV)

ITER: Out of all advanced divertor configurations, only the X-divertor is physically achievable

- Divertor configurations on ITER
  - Snowflake divertor – exceeds coil currents [1]
  - Super X divertor – geometry changes [2]
  - X divertor possible [2]

ITER: Physical differences on ITER necessitate a more integrated control approach (MPC)

- Poloidal field coils are far away from the plasma, flux effects are more coupled
- No separate set of divertor coils
- Easy to run into coil current constraints
- System is not strictly controllable
  - 12 PF coils but only 11 independent coil circuits
  - 31 shape pts + $I_p + \Psi_{br} + 6$ divertor variables = 39 outputs
  - Plus constraint set (35 additional variables)

### Activated Constraints

<table>
<thead>
<tr>
<th>Coil #</th>
<th>PF Coils</th>
<th>Outputs</th>
</tr>
</thead>
<tbody>
<tr>
<td>PF1</td>
<td>$&lt; 48 \text{kA}$</td>
<td>$&lt; 1.5 \text{kV}$</td>
</tr>
<tr>
<td>PF2</td>
<td>$&lt; 55 \text{kA}$</td>
<td>$&lt; 1.5 \text{kV}$</td>
</tr>
<tr>
<td>PF3</td>
<td>$&lt; 55 \text{kA}$</td>
<td>$&lt; 1.5 \text{kV}$</td>
</tr>
<tr>
<td>PF4</td>
<td>$&lt; 55 \text{kA}$</td>
<td>$&lt; 1.5 \text{kV}$</td>
</tr>
<tr>
<td>PF5</td>
<td>$&lt; 52 \text{kA}$</td>
<td>$&lt; 1.5 \text{kV}$</td>
</tr>
<tr>
<td>PF6</td>
<td>$&lt; 52 \text{kA}$</td>
<td>$&lt; 1.5 \text{kV}$</td>
</tr>
<tr>
<td>CS1U</td>
<td>$&lt; 45 \text{kA}$</td>
<td>$&lt; 1.5 \text{kV}$</td>
</tr>
<tr>
<td>CS1L</td>
<td>$&lt; 45 \text{kA}$</td>
<td>$&lt; 1.5 \text{kV}$</td>
</tr>
<tr>
<td>CS2U</td>
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<td>$&lt; 3.0 \text{kV}$</td>
</tr>
<tr>
<td>CS2L</td>
<td>$&lt; 45 \text{kA}$</td>
<td>$&lt; 1.5 \text{kV}$</td>
</tr>
<tr>
<td>CS3U</td>
<td>$&lt; 45 \text{kA}$</td>
<td>$&lt; 1.5 \text{kV}$</td>
</tr>
<tr>
<td>CS3L</td>
<td>$&lt; 45 \text{kA}$</td>
<td>$&lt; 1.5 \text{kV}$</td>
</tr>
</tbody>
</table>

Red cells affect the control optimization
ITER: MPC optimizes the control inputs over a finite horizon, subject to constraints

- Quadratic cost on the output errors and control actuation
  \[ J_k = \sum_{i=1}^{N} \left[ (y_{k+i} - r_{k+i})^T Q_i (y_{k+i} - r_{k+i}) + u_{k+i-1}^T R_i u_{k+i-1} \right] \]

- Use dynamics model to predict future outputs
  \[
  \begin{align*}
  x_{k+1} &= A x_k + B u_k \\
  y_{k+1} &= C x_{k+1}
  \end{align*}
  \]

- After substitution, obtain convex cost function in standard quadratic-program form
  - Solve via `mpcqp solver` in MATLAB

\[
J_k = \hat{U}^T H \hat{U} + 2 f^T \hat{U} + J_\theta
\]

\[
\hat{U} := \begin{bmatrix}
  u_k \\
  u_{k+1} \\
  \vdots \\
  u_{k+N-1}
\end{bmatrix}
\]
ITER: MPC is computationally intensive, but is expected to be feasible for real-time

- MPC can be fast (3-7 ms) [1], could be used in real-time
- Several tricks for speeding up simulation
  - Truncated prediction model
    - Neglects vacuum vessel currents
    - $(N \times 13)$ versus $(N \times 163)$
- Move blocking
  - Reduces the number of optimization variables
  - Geometrically scaling block sizes

**ITER:** X-divertor can be achieved while satisfying constraint set, \( I_p = 10 \text{ MA} \)
ITER: large changes to the secondary x-point location can be realized with minimal impact on the primary x-point and shape
DIII-D: the Infrared TV diagnostic can be used to identify snowflake x-points and better constrain the equilibrium reconstruction – P.J. Vail

- IRTV diagnostic measures heat flux on the divertor plates
- Predicted heat flux of the snowflake equilibrium reconstruction does not match IRTV
  - Opportunity for IRTV to provide additional info to reconstruction algorithm

- **Approach**
  - Analytical model [1]: x point locations --> heat flux
  - ML regression tree: heat flux --> x point locations
  - Use predicted x-points to constrain equilibria

- Constrained equilibria match measured heat flux better

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• ML Predictions
  • 17 shots with 25 time slices each
  • ~ 1cm error on the testing data set

• How do the profiles in the heat-flux constrained equilibrium differ?

• ~20% difference in edge current. Further studies to perform this analysis across the database and quantify.
NSTXU: for overall divertor performance, the snowflake divertor must work well with the particle exhaust mechanism - P.J. Vail

- Divertor functions: power exhaust AND particle exhaust
- Does the snowflake divertor work well with conventional particle exhaust (cryopump)?
  - How to optimally place cryopump?

- Analytical model for snowflake power flux
  - Diffusion eqn solved in 2 separate domains, characterizes better than a standard divertor with large flux expansion

**NSTXU: An optimal cryopump location allows for full power and particle exhaust over a range of snowflakes** - P.J. Vail [1]

- Heat flux profile directly related to particle flux profile [2]

\[ \Gamma_{\perp}^{div} = q_{\perp}^{div} / \gamma T_e \]

- Assumptions
  - 24 kL/s volumetric pump rate for liquid helium cooled cryopump
  - 10 MW (20 Torr-L/s) of neutral beam heating
  - Gives inlet pressure condition[1,3]: \( P > 0.83 \) mTorr

Summary

- Developing multiple analysis and control tools to improve performance of advanced divertor configurations.

- Snowflake divertor control on NSTX can be achieved with high degree of control. Highlights need for online model changes.

- Model predictive control on ITER
  - Large changes in the divertor field geometry can be obtained within the limits of physical constraints.
  - It may be possible to create and test the x-divertor on ITER.

- IRTV can be used as a diagnostic to improve snowflake equilibrium reconstructions on DIII-D.

- Improved UEDGE simulations guide the design of optimal cryopump locations for NSTXU snowflakes.

**Future work**

- Perform larger analyses of IRTV edge current predictions.
- Implement online model changes for NSTX in order to control ramp-up scenarios (M.D. Boyer, P.J. Vail).