Stellarator Phase Space Exploration with DESC

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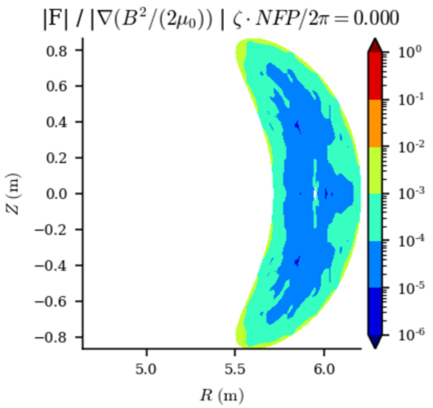
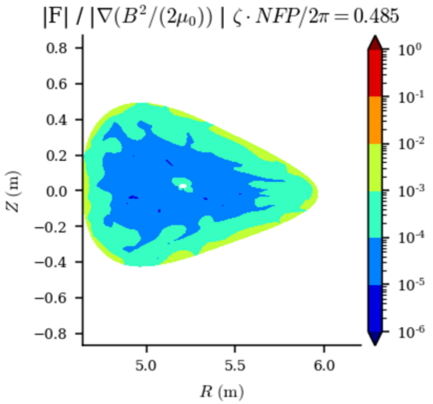
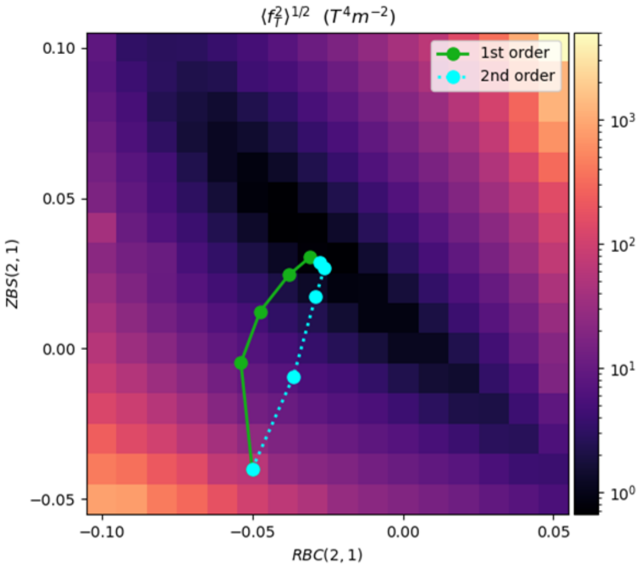
DESC is the ideal tool for stellarator equilibrium solving and optimization

Accurate Equilibria

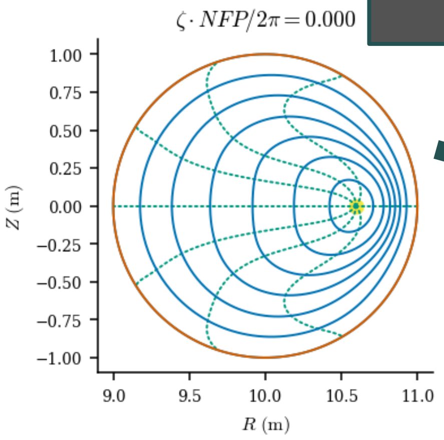




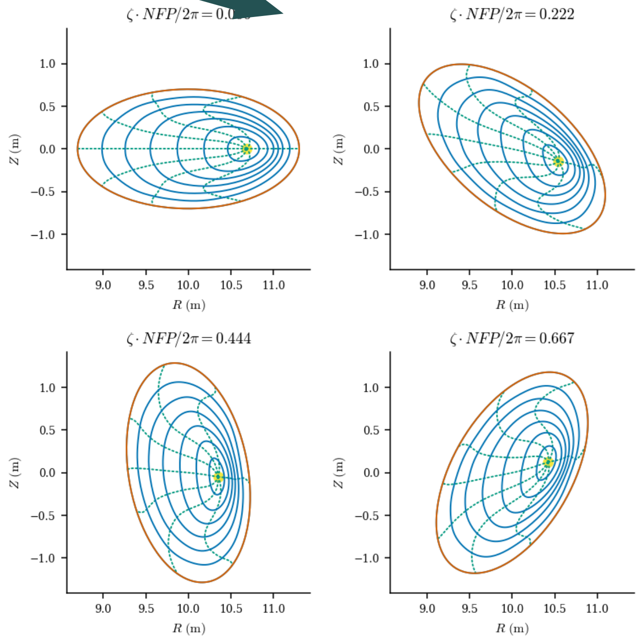
Fast Optimization



Phase-Space Connections



perturb tokamak into stellarator



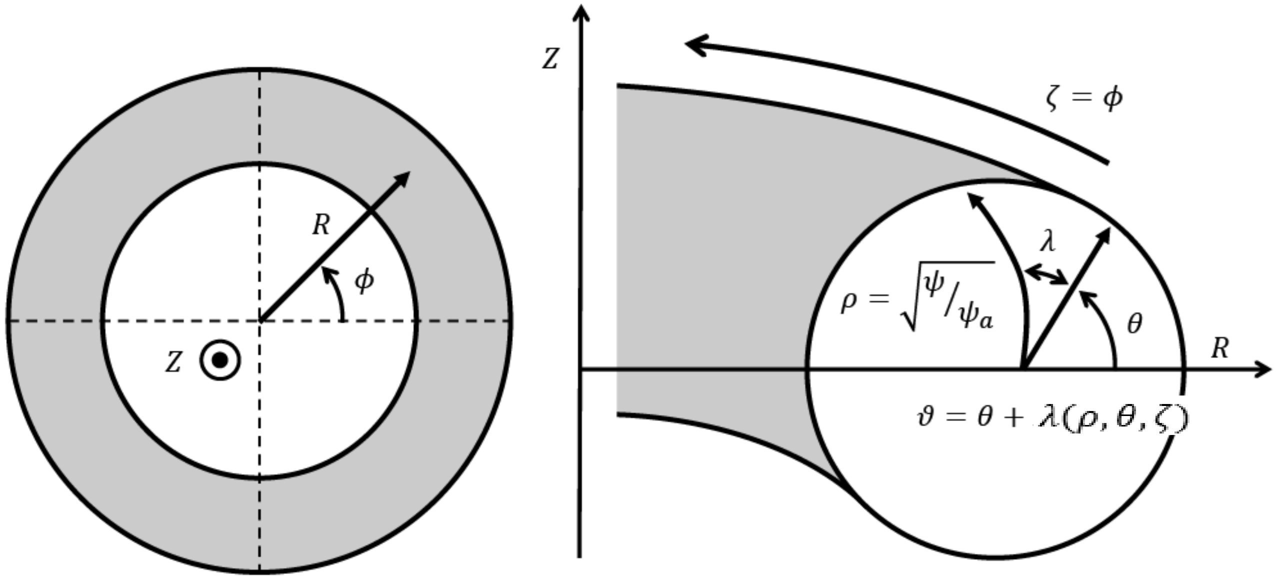
Stellarator Equilibrium and Optimization - DESC

- 3D Ideal MHD Equilibrium Code
- Assumes Nested Flux Surfaces
- Inverse Equilibrium Problem
- Minimizes Force Error Directly

$$\boldsymbol{F} = \boldsymbol{J} \times \boldsymbol{B} - \nabla p = 0$$

- Pseudospectral Code

3D Spectral Representation of $\mathbf{x} = (R, \lambda, Z)$ using Fourier-Zernike Basis



(Dudt and Kolemen 2020)

What do we want when we design a stellarator?

The most basic: A non-axisymmetric MHD equilbrium

Non-axisymmetric Magnetic Fields

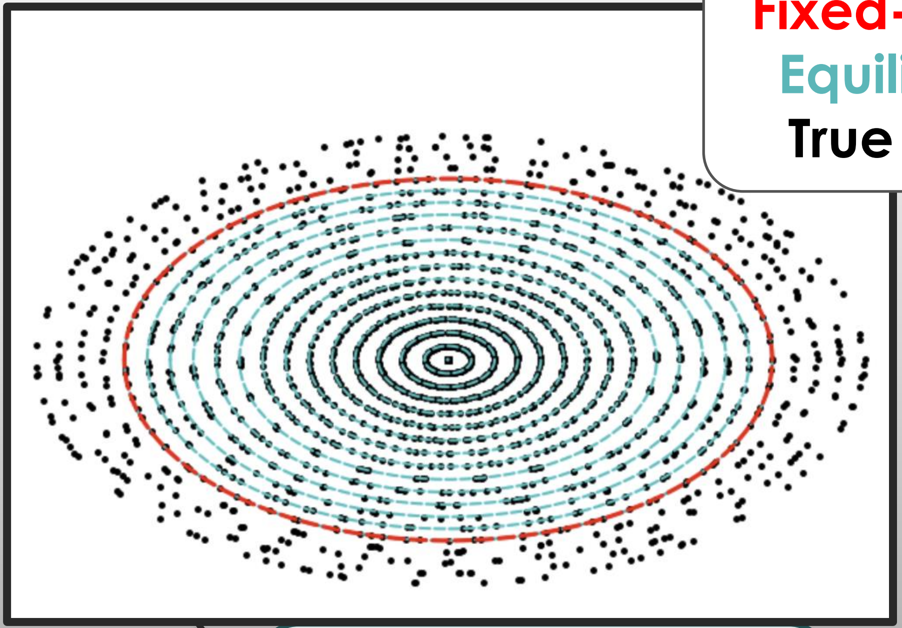
We want nested flux surfaces for confinement

Non-axisymmetric Magnetic Fields

Nested Flux Surfaces

How can we know we will actually get nested flux surfaces?

Non-axisymmetric Magnetic Fields

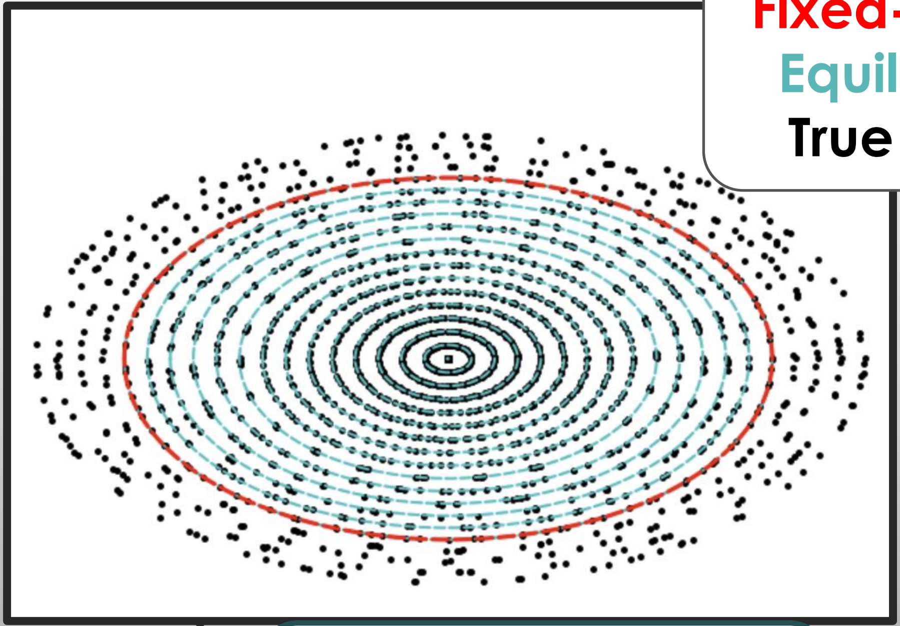


Fixed-Boundary input
Equilibrium Solution
True Vacuum Field

Nested Flux Surfaces

How can we know we will actually get nested flux surfaces?

Non-axisymmetric Magnetic Fields



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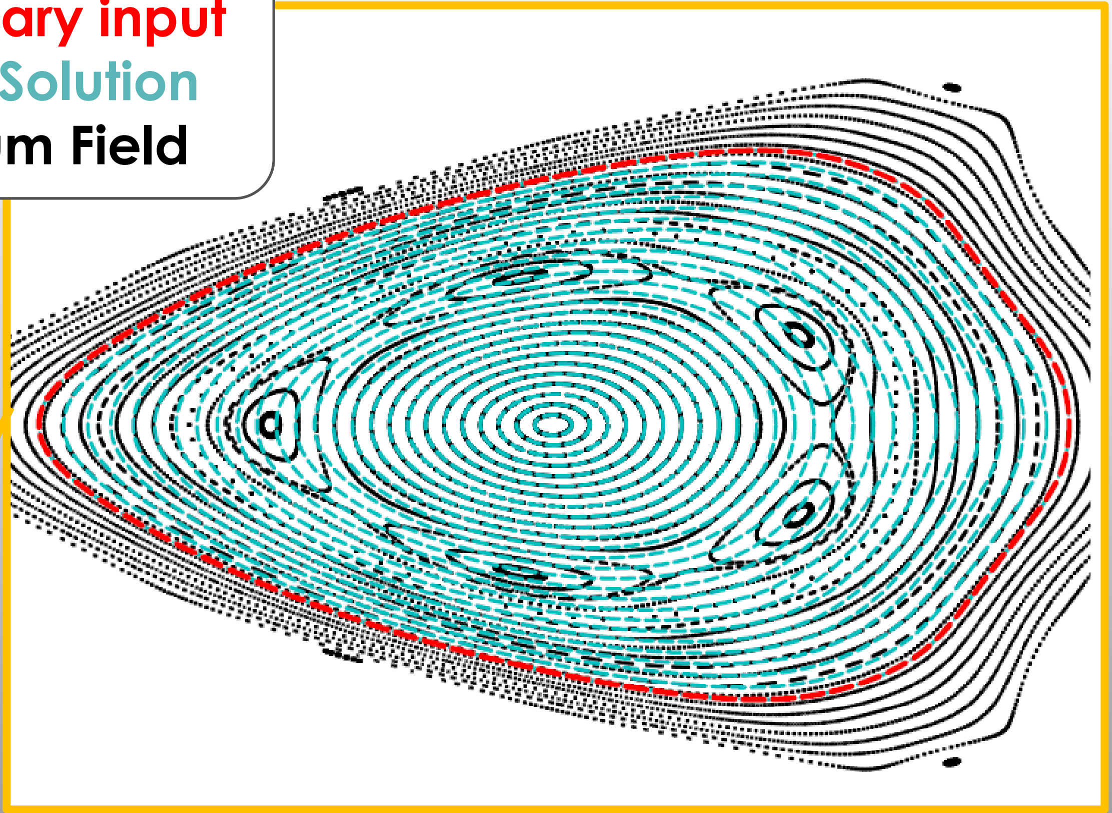
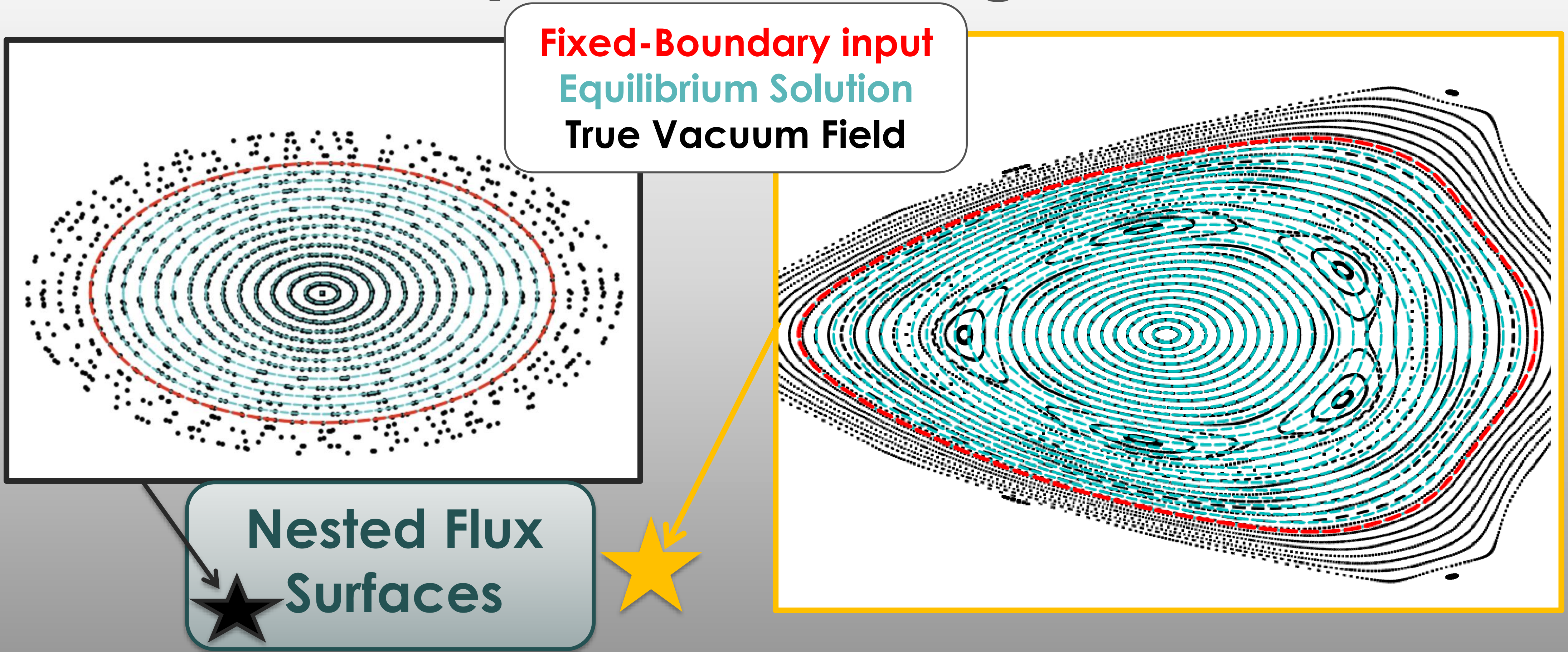


Nested Flux Surfaces

Non-axisymmetric Magnetic Fields

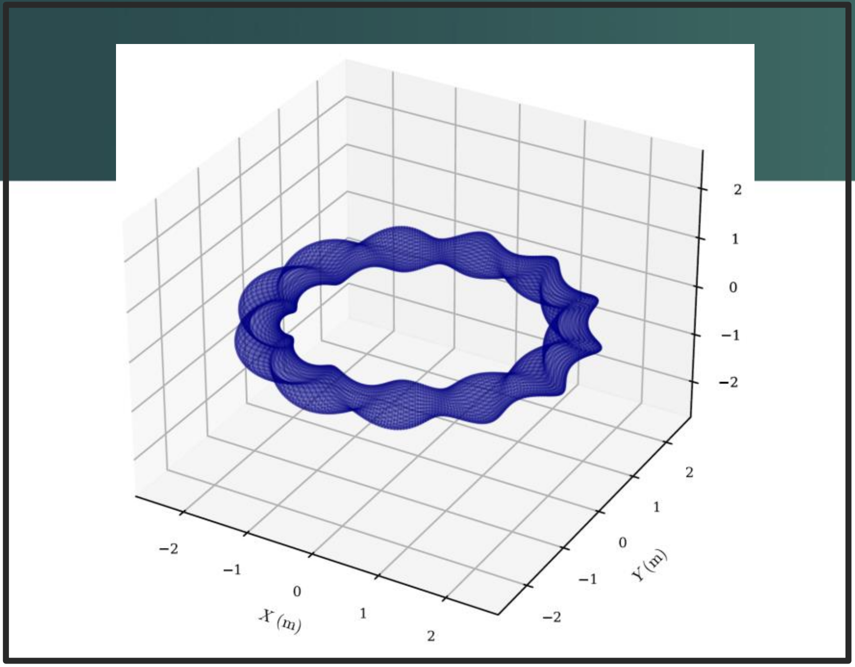


Define this subspace!

Plasma Control

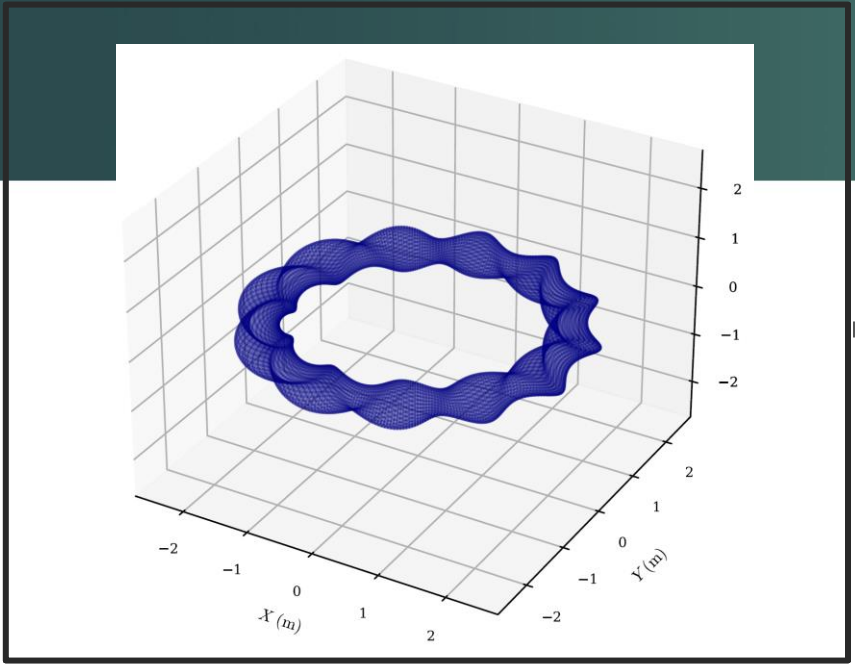
Poincare Boundary Condition

Conventional Last-Closed-Flux-Surface Boundary Condition



INPUT: R,Z of LCFS at ρ=1

Conventional Last-Closed-Flux-Surface Boundary Condition



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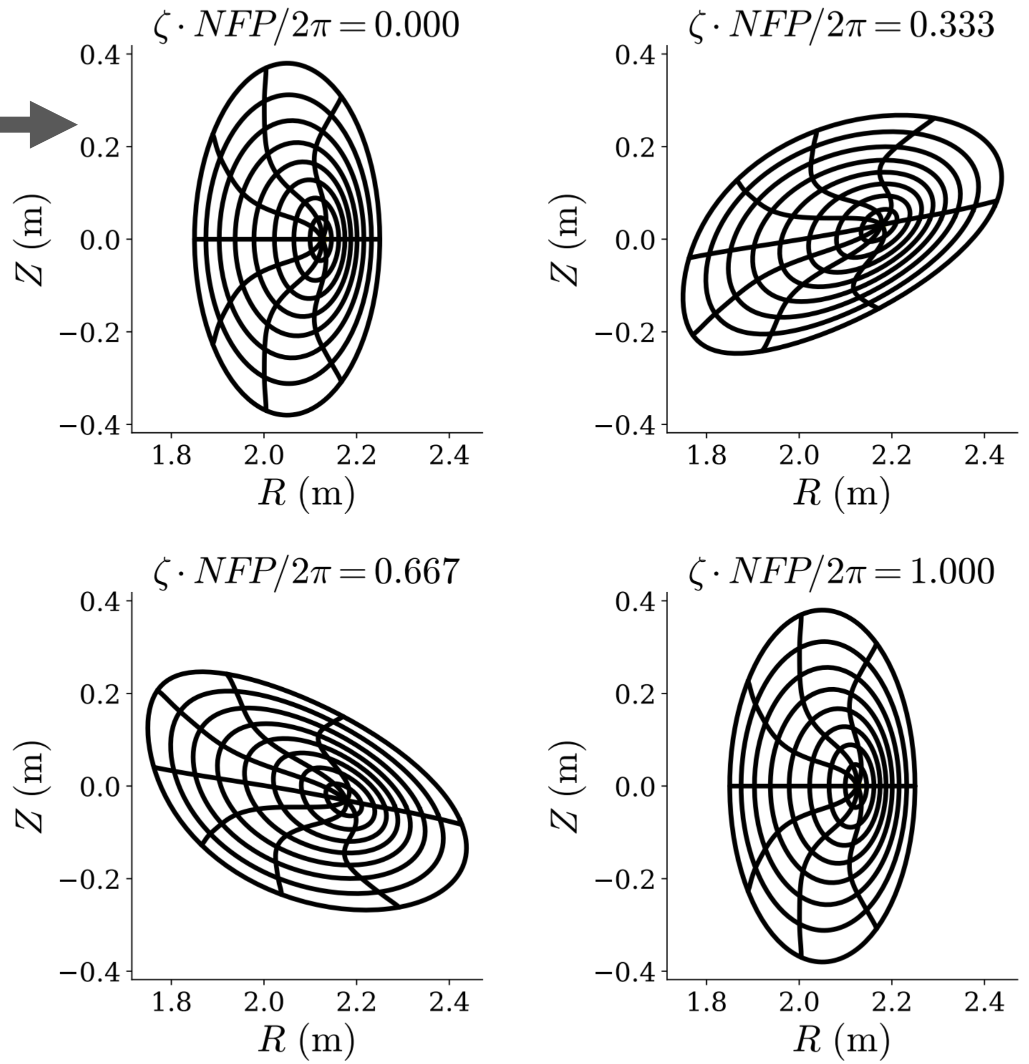
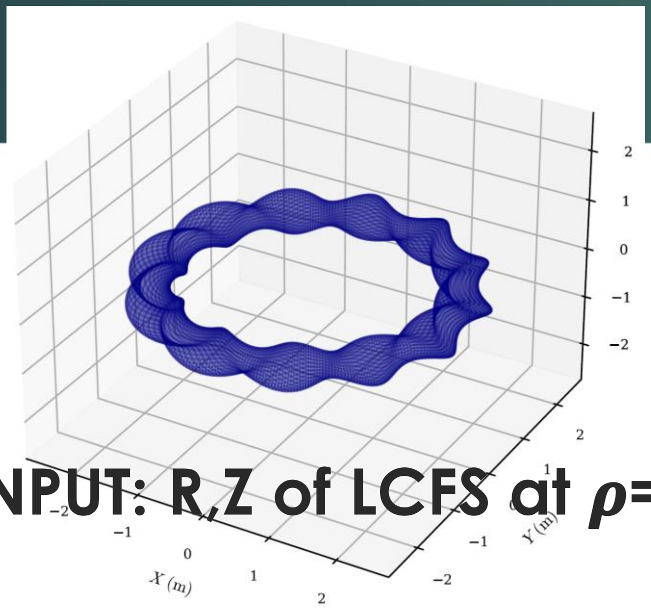


Poincare Section could better parameterize stellarator design space



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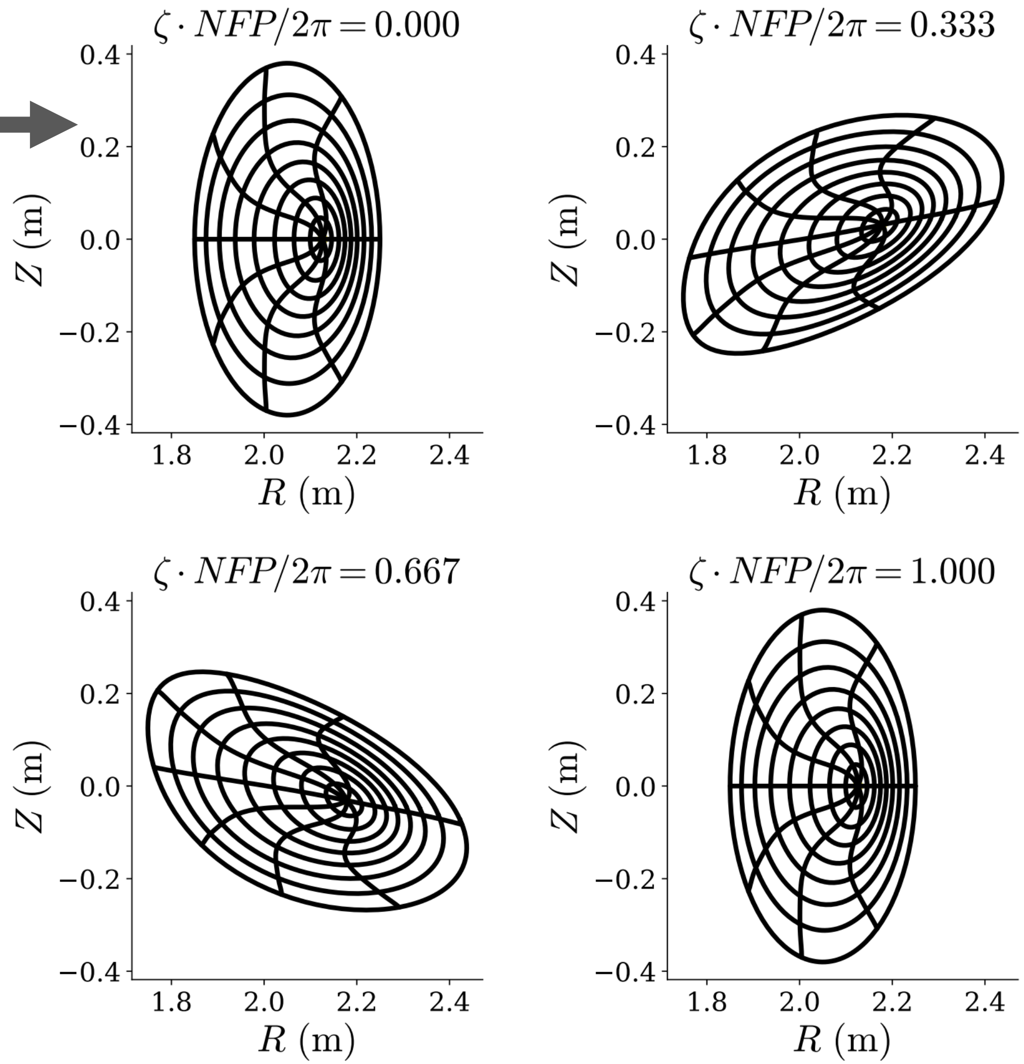
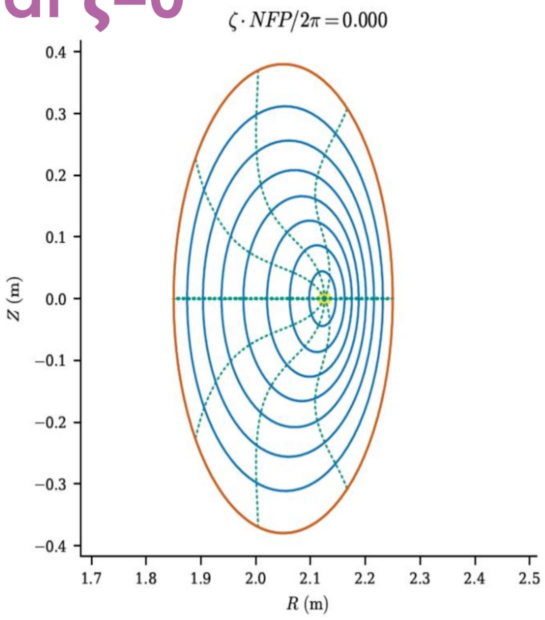
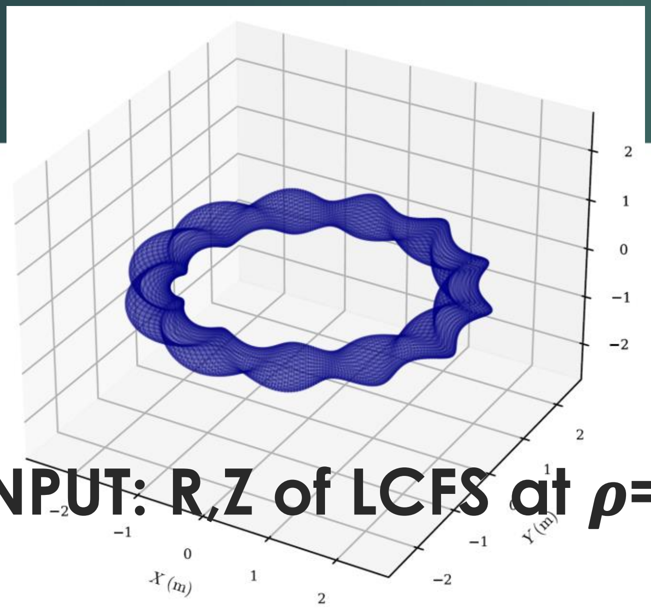


INPUT: R,Z, λ of Poincare XS at ζ=0



Plasma Control

Poincare Section could better parameterize stellarator design space



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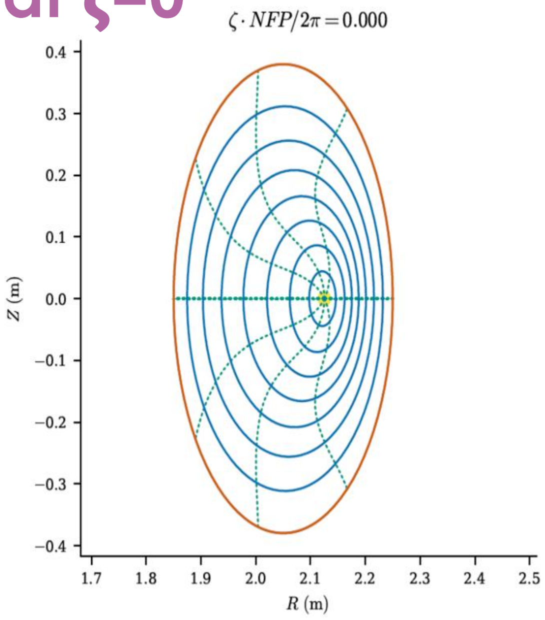

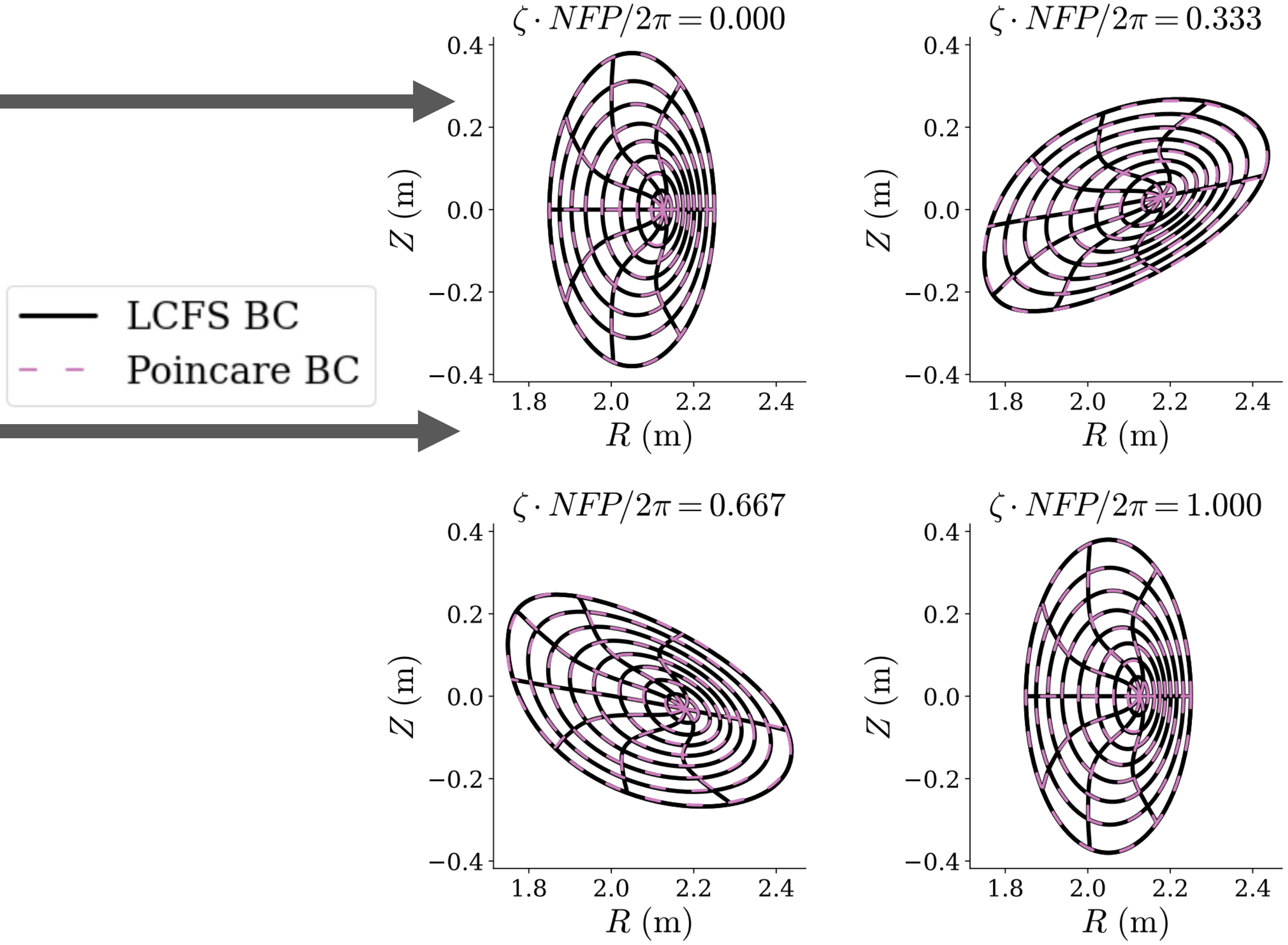

Plasma Control

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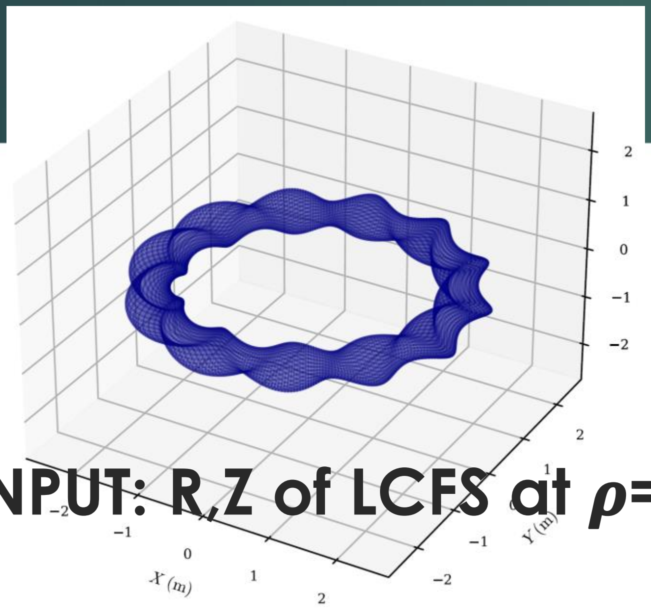

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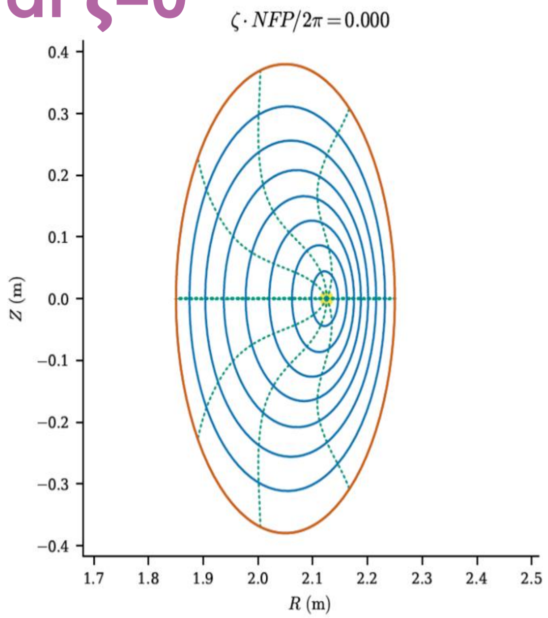
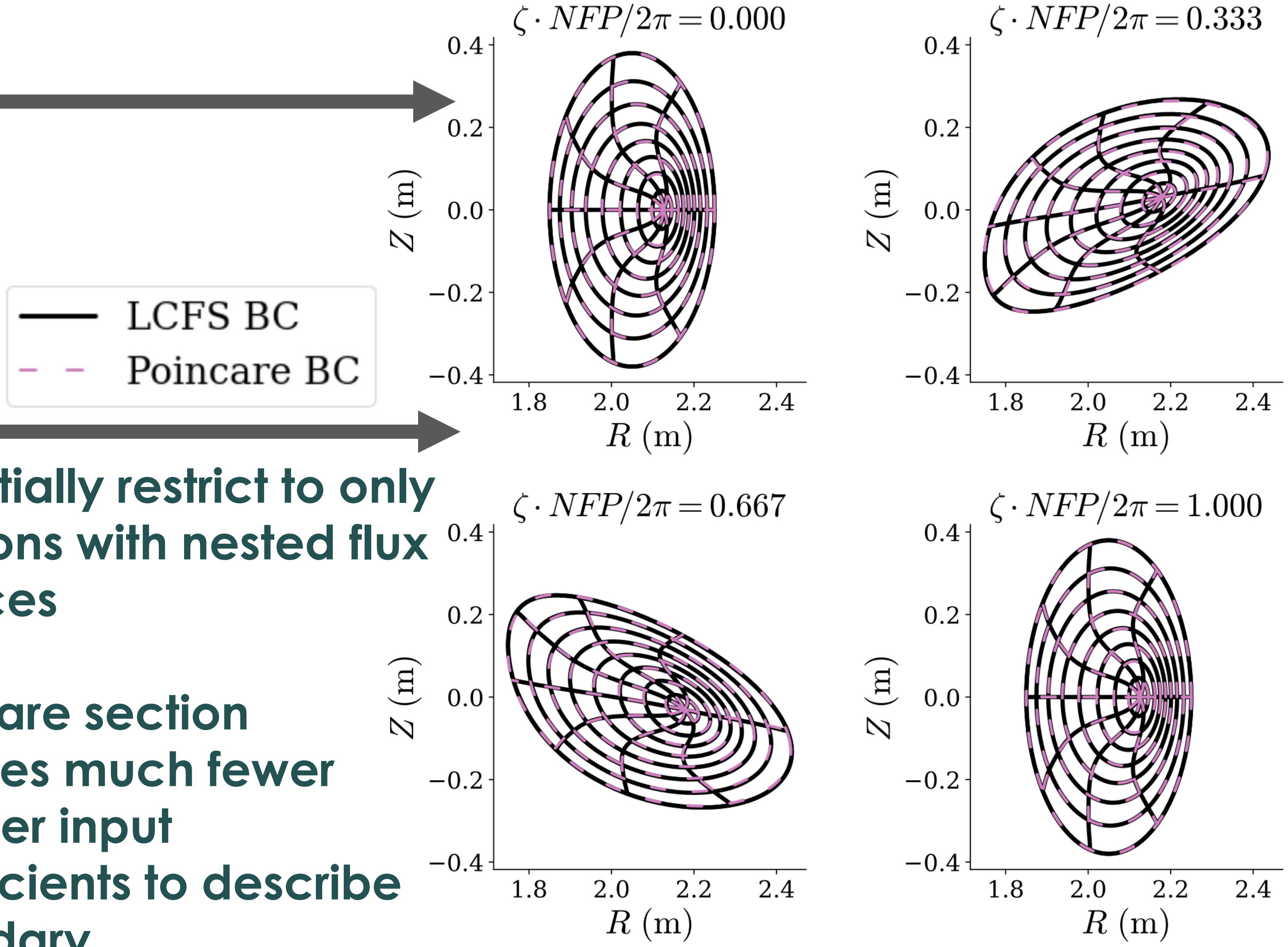




- **Potentially restrict to only solutions with nested flux surfaces**
- **Poincare section requires much fewer number input coefficients to describe boundary**

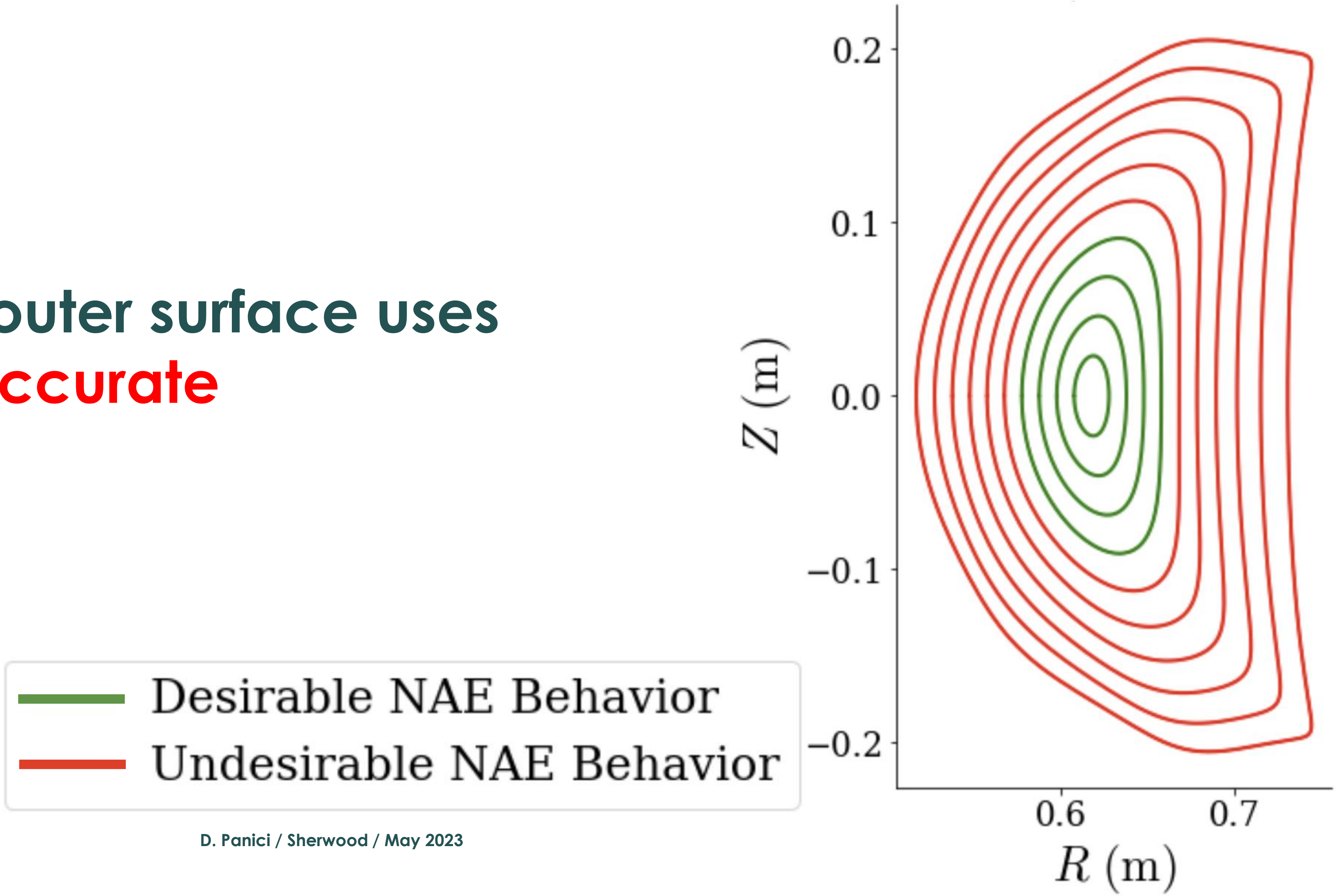
Plasma Control

Near-Axis Constrained Equilibria

Given a NAE solution, how do we find a global MHD solution?

Evaluating and fixing outer surface uses NAE where it is **least accurate**

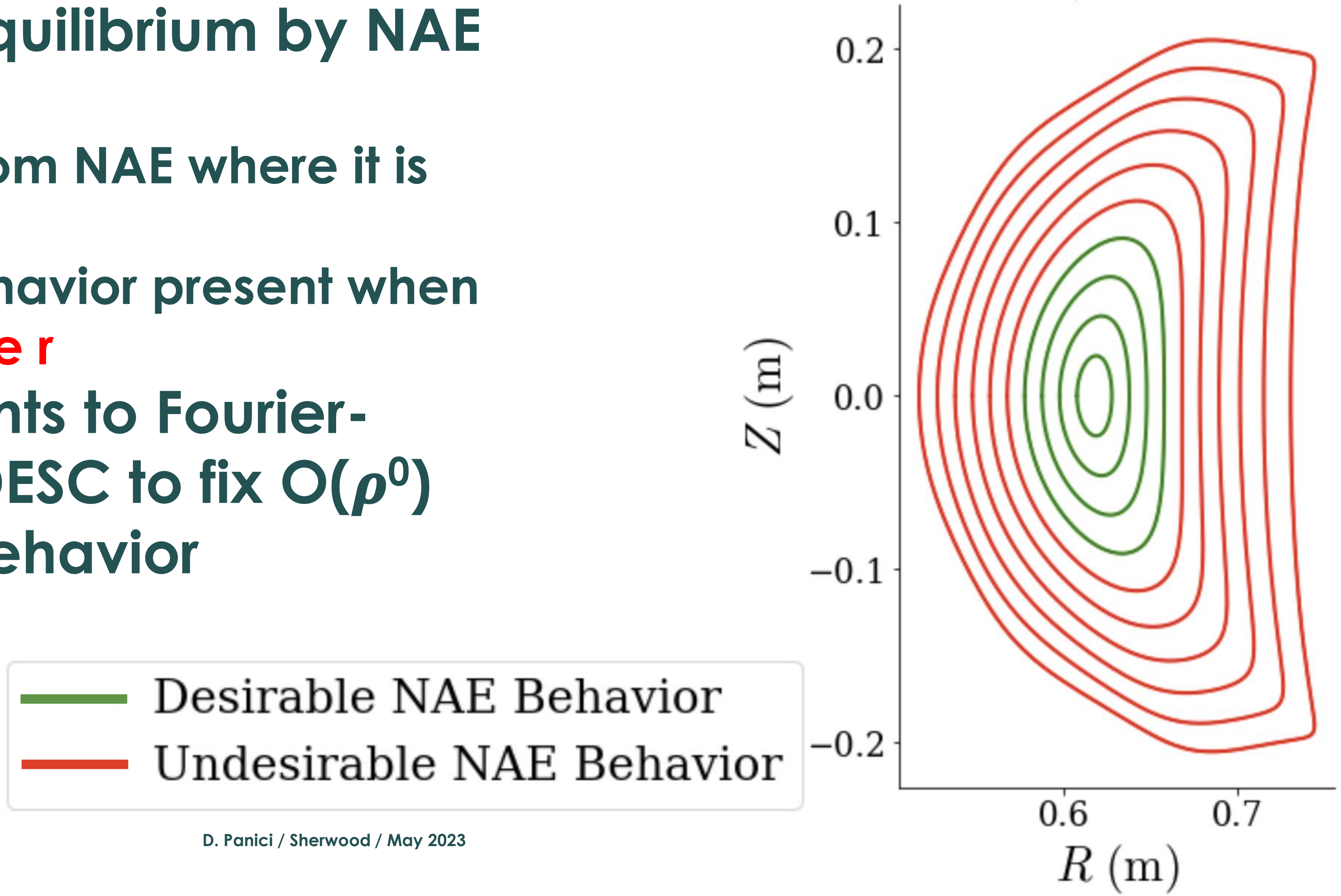
NAE equilibrium evaluated at r =0.1



Near-Axis Expansion (NAE) Constraints in DESC (with E. Rodriguez)

- Constrain global equilibrium by NAE behavior as $\rho \to 0$
  - Use information from NAE where it is most valid
  - Avoid singular behavior present when evaluating at large r
- Map NAE coefficients to Fourier-Zernike modes of DESC to fix $O(\rho^0)$ (axis) and $O(\rho^1)$ behavior

NAE equilibrium evaluated at r =0.1



O(ρ^0) (axis) Constraint in DESC

Given a NAE axis as Fourier series in cylindrical toroidal angle ϕ:

$$R = R_0 + \sum_{n=1}^{N} (R_n^C \cos m\phi + R_n^S \sin m\phi)$$

$$Z = \sum_{n=1}^{N} (Z_n^C \cos m\phi + Z_n^S \sin m\phi)$$

There exists a simple, linear mapping to the DESC Fourier-Zernike basis:

NAE Axis Coefficients

$$R_n^{C/S} = \sum_{k=0}^{\infty} (-1)^k R_{2k,0,\pm|n|}$$

$$Z_n^{C/S} = \sum_{k=0}^{\infty} (-1)^k Z_{2k,0,\pm|n|}$$

DESC Fourier-Zernike Coefficients

$O(\rho^1)$ NAE Constraint in DESC

- Given $O(\rho)$ R,Z position flux surface from the NAE:

$$\mathbf{r} \approx \mathbf{r}_0(\phi) + \rho R_1 \hat{\mathbf{R}} + \rho Z_1 \hat{\mathbf{Z}}$$

where

$$R_1 = \mathcal{R}_{1,1}(\phi)\cos\theta + \mathcal{R}_{1,-1}(\phi)\sin\theta$$

$$Z_1 = Z_{1,1}(\phi)\cos\theta + Z_{1,-1}(\phi)\sin\theta$$

There again exists a simple, linear mapping to the DESC Fourier-Zernike basis:

NAE Coefficients

$$\mathcal{R}_{1,1,n} = -\sum_{k=1}^{M}(-1)^k k R_{2k-1,1,n},$$

$$\mathcal{R}_{1,-1,n} = -\sum_{k=1}^{M}(-1)^k k R_{2k-1,-1,n},$$

DESC Fourier-Zernike Coefficients

(Identical expressions for Z)

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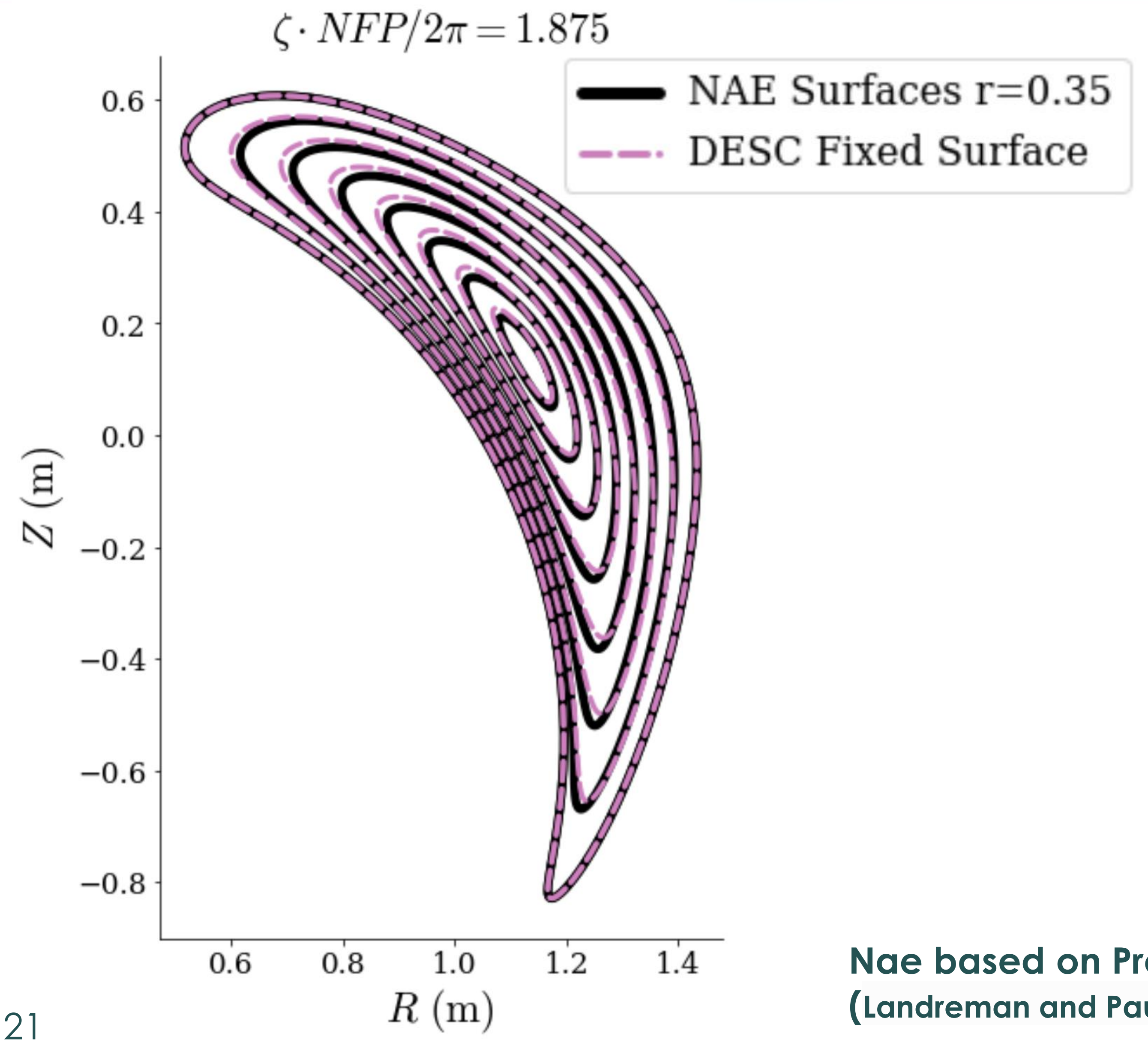
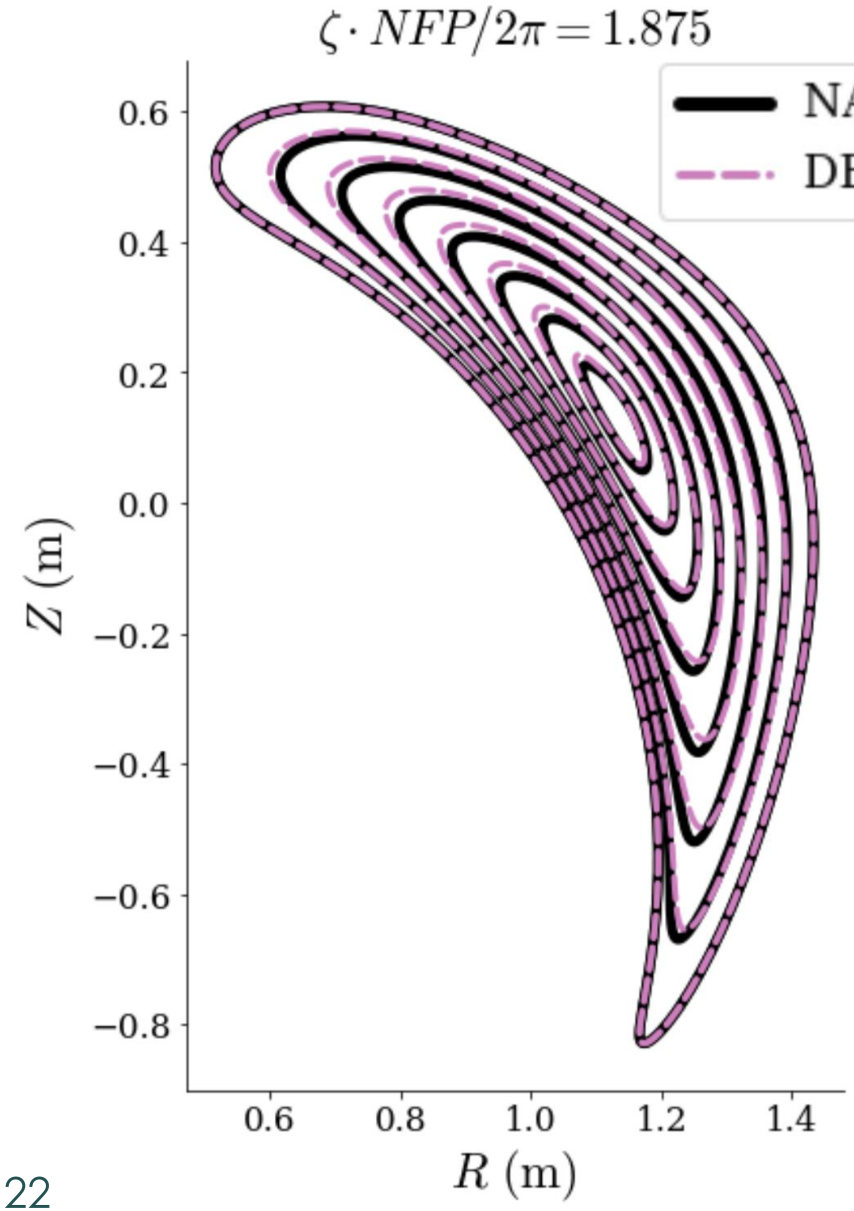
Fixed-Boundary Solve from NAE surface

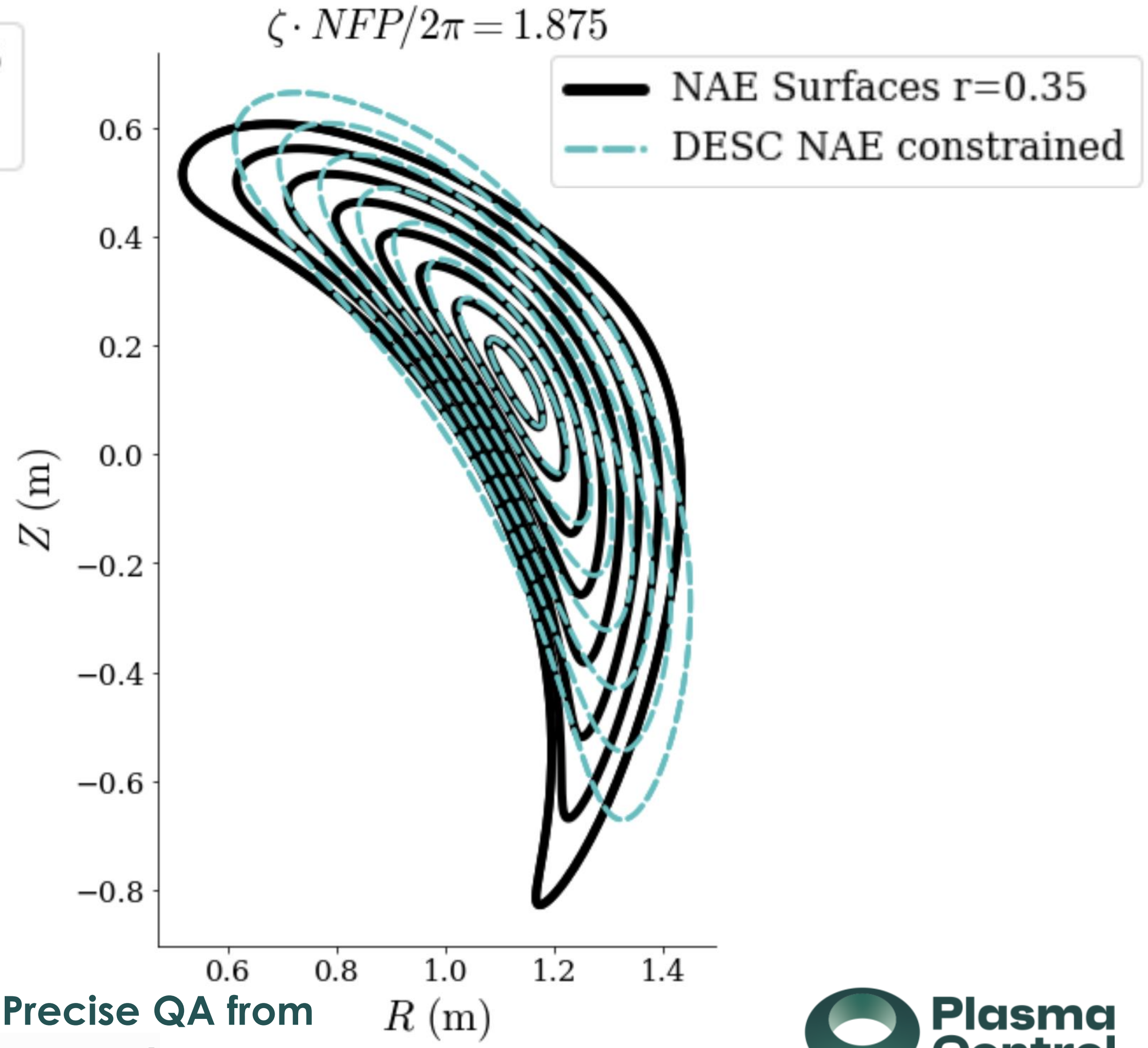


Nae based on Precise QA from
(Landreman and Paul 2022)

Plasma Control

NAE Constraint in DESC - Solved Equilibrium Agrees with NAE surfaces NEAR-AXIS, unlike Surface Solve





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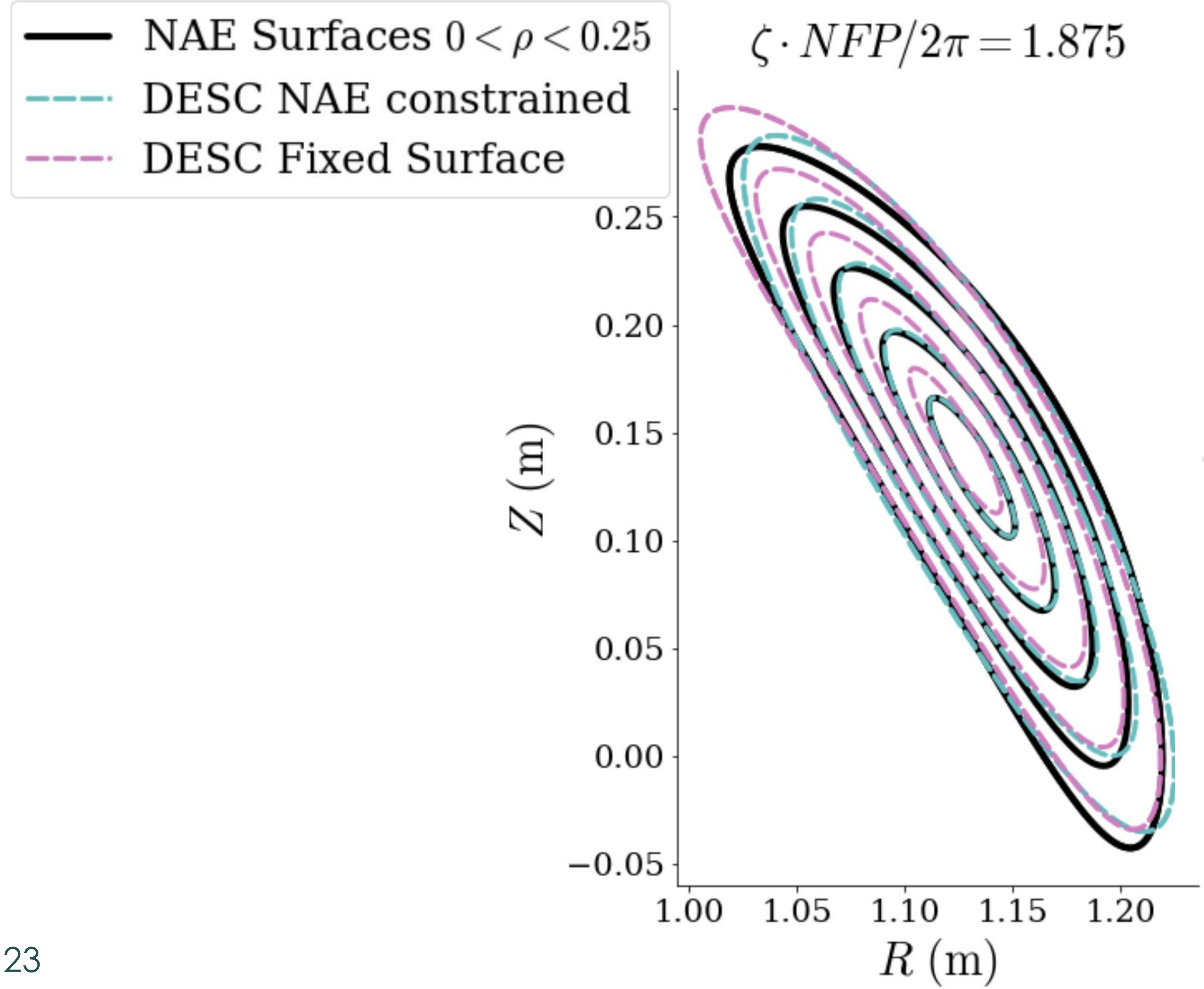

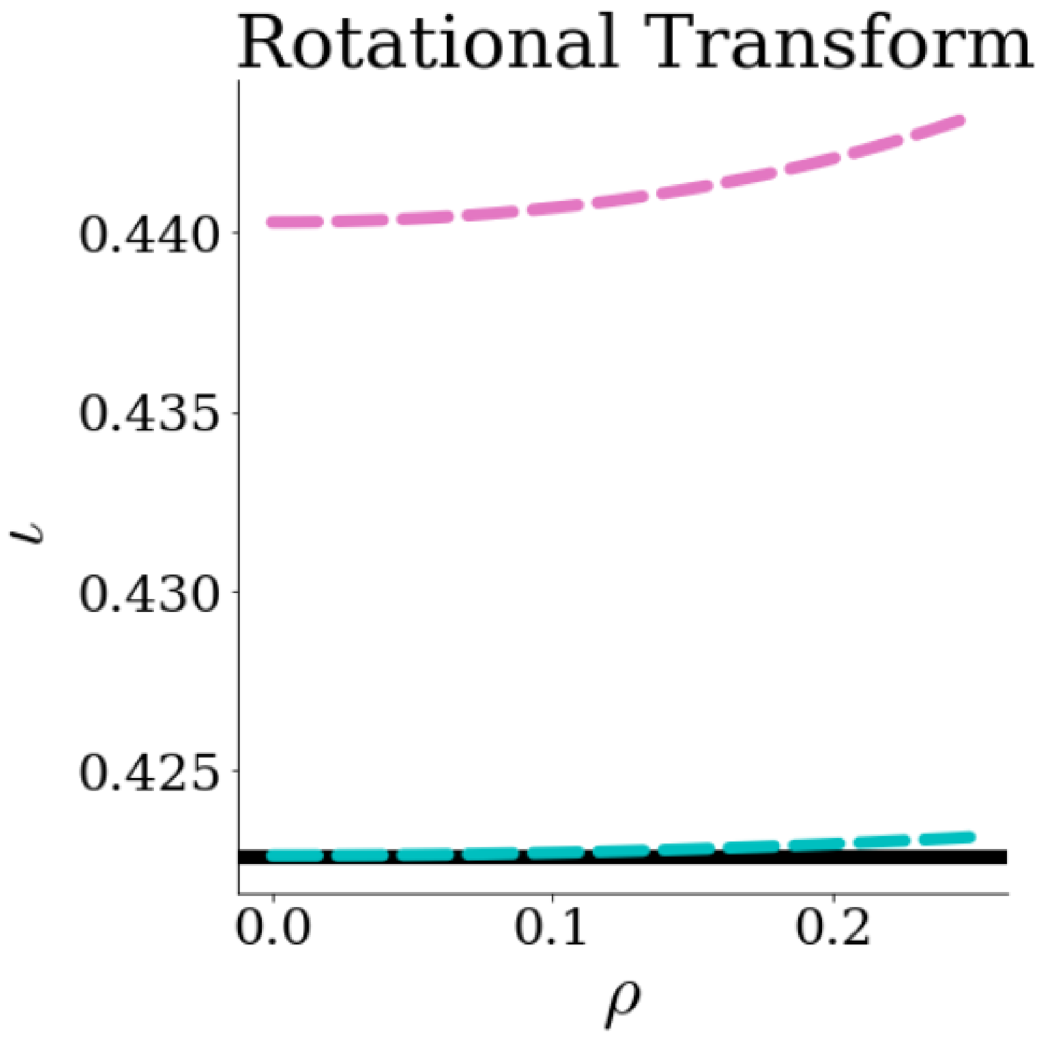

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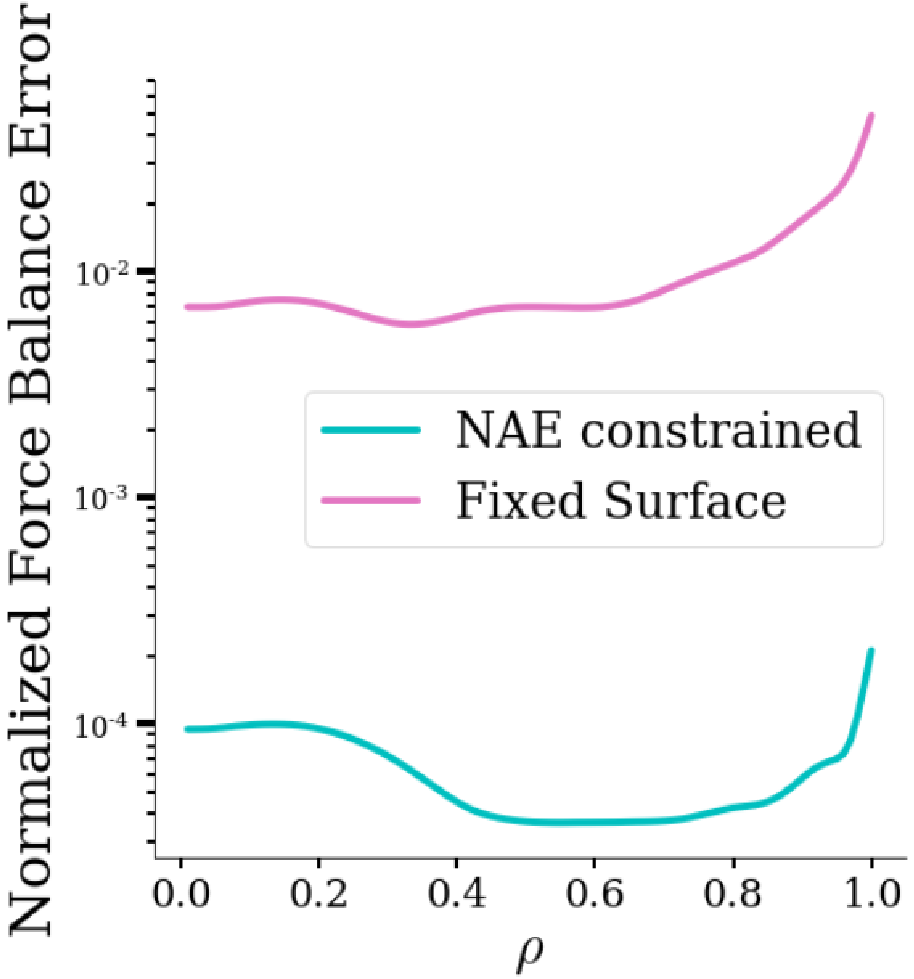
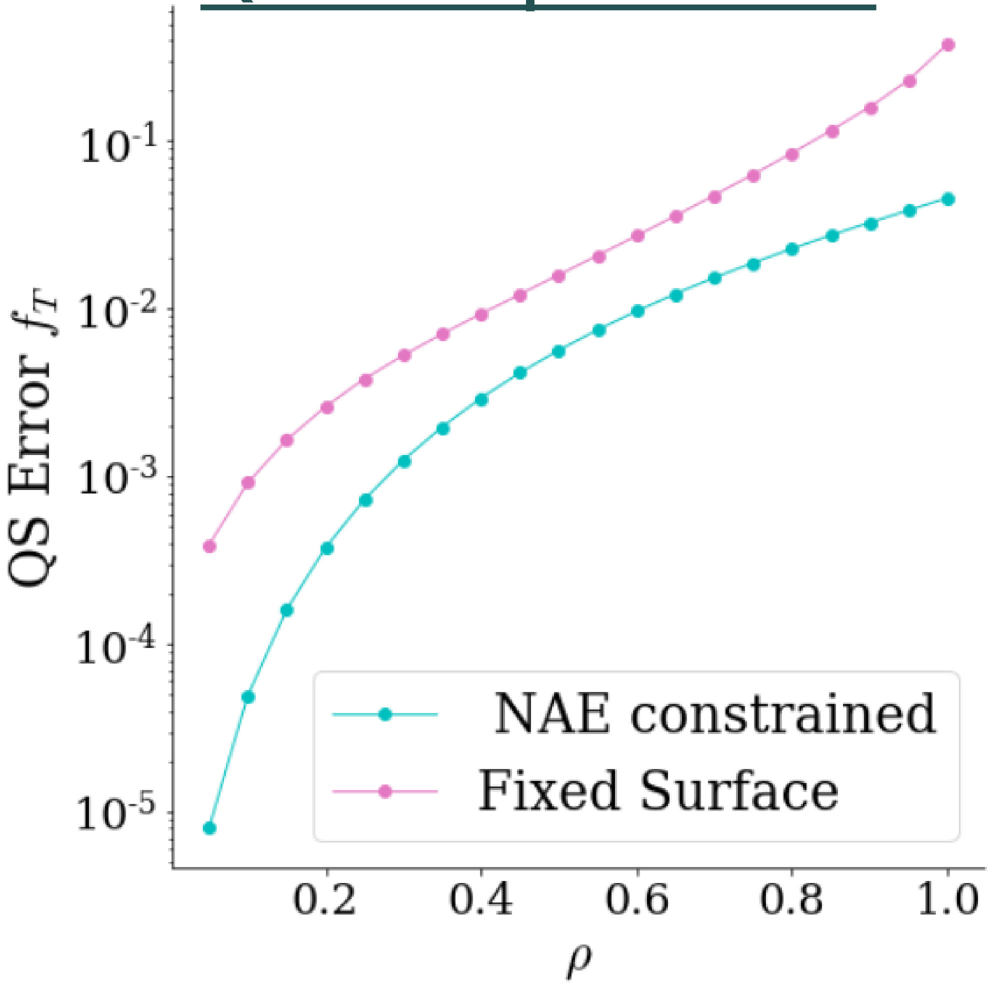
NAE Constraint in DESC - Lower Force Error and Retains QS near-axis

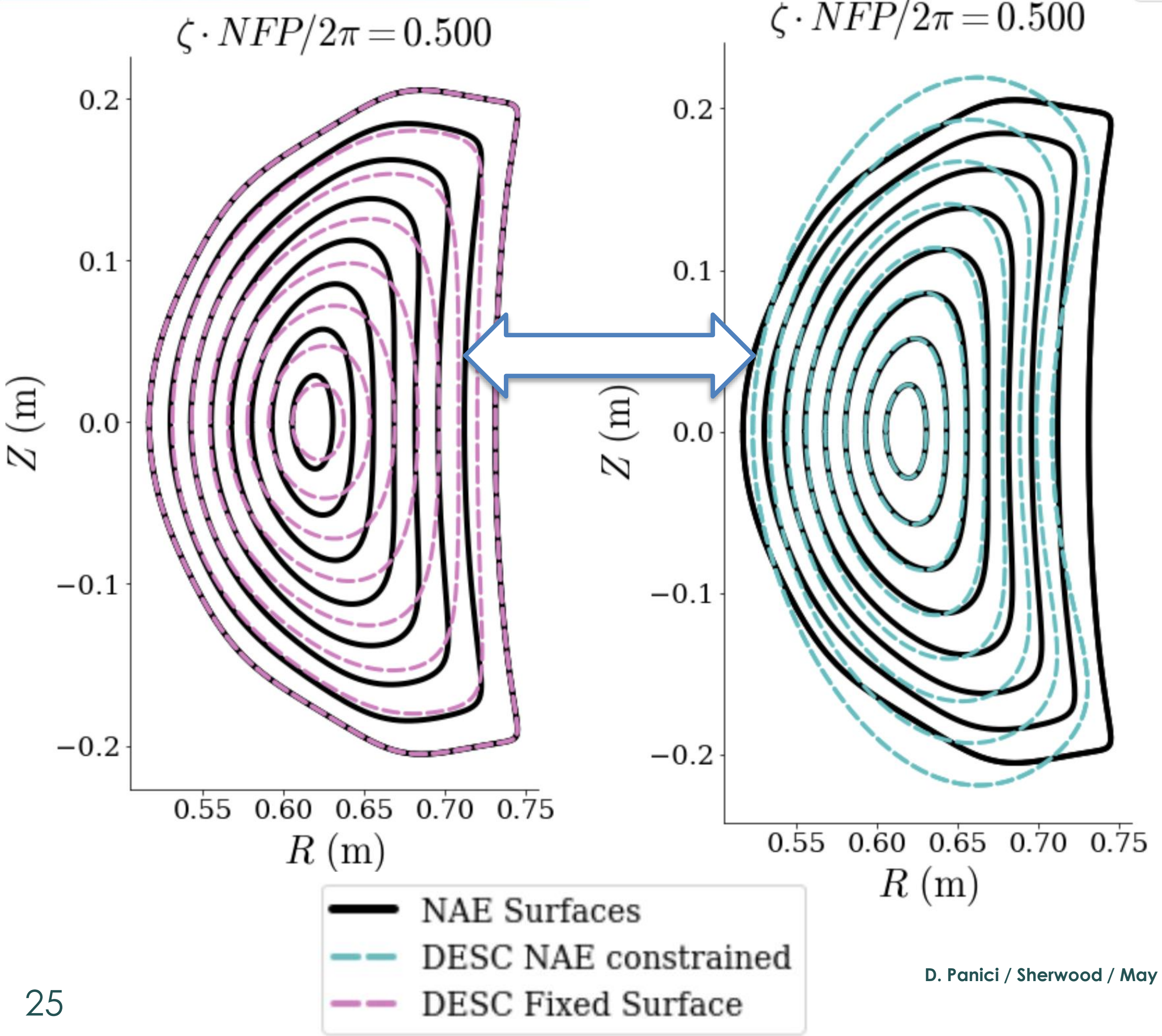
Force Error

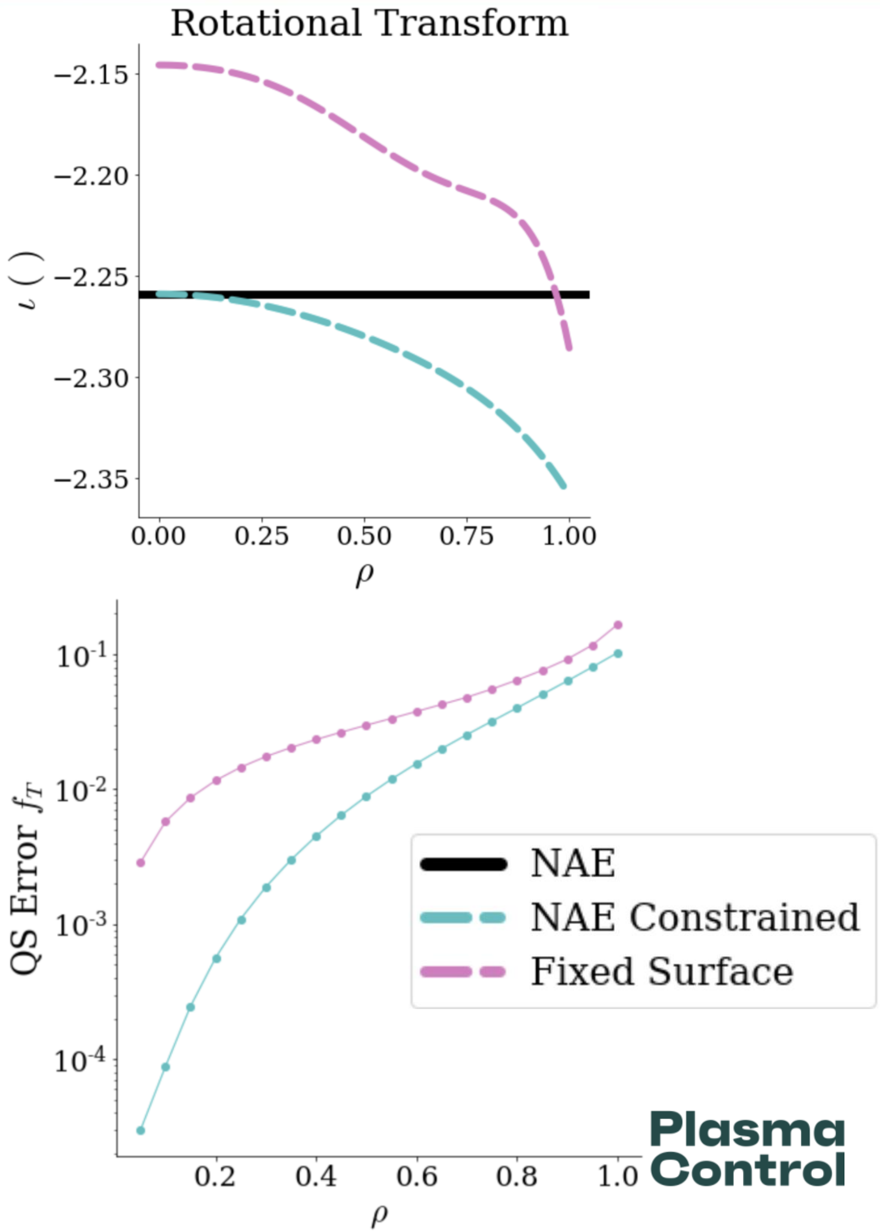


QS Error Triple Product



$O(\rho^1)$ Constraint in DESC - Example Solve where Fixed Surface Struggles (Example From E. Rodriguez)





We want more than just nested flux surfaces

Nested Flux Surfaces

We want more than just nested flux surfaces

Nested Flux Surfaces

Omnigenous Fields

We want more than just nested flux surfaces

Nested Flux Surfaces

Omnigenous Fields

Define this subspace!

Quasisymmetric Fields

Omnigenous Phase Space Definition

Omnigenous magnetic fields

Particles in omnigenous magnetic fields have no net radial drifts

Quasi-Isodynamic (QI) magnetic fields = omnigenous magnetic fields with constant |B| contours that close poloidally

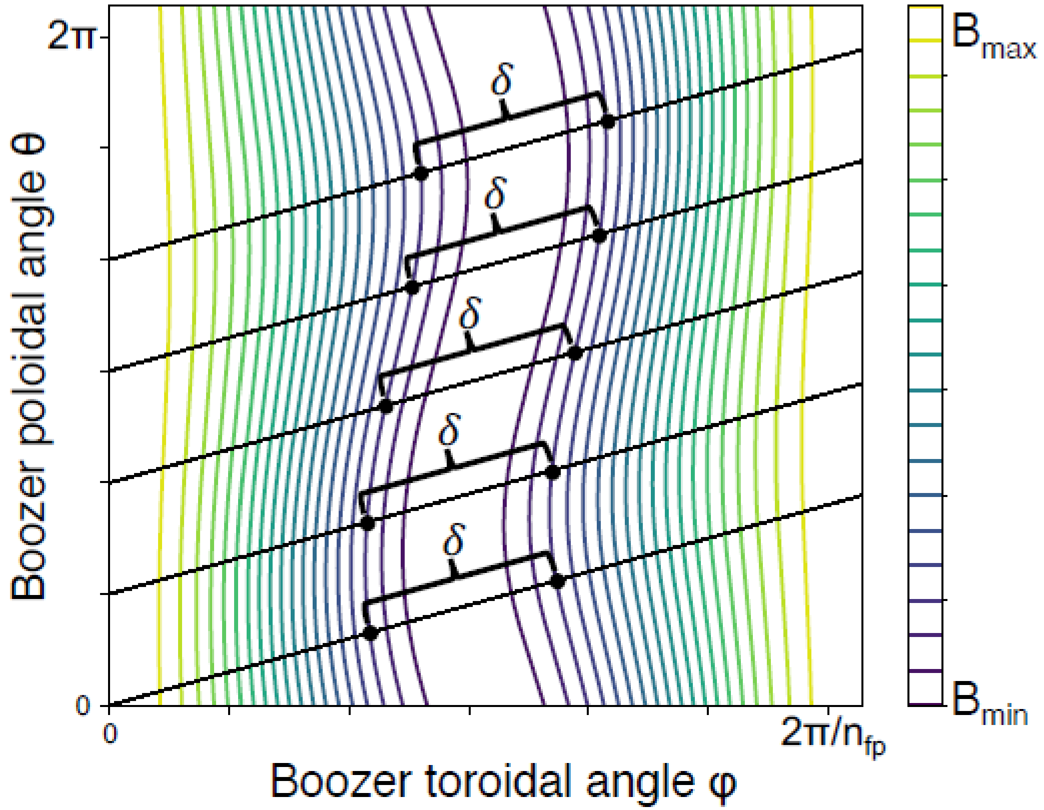
<u>Conditions for Omnigenity:</u>

- B_{max} is a straight contour in Boozer coordinates
- Constant "bounce distance" δ between consecutive points of equal B on each field line α

$$\delta = \sqrt{\Delta\theta_B^2 + \Delta\zeta_B^2} \propto \Delta\zeta_B \qquad \frac{\partial\delta}{\partial\alpha} = 0$$

<u>Model Assumptions:</u>

- Single magnetic well per field period
- No stellarator symmetry assumption



(Goodman 2022)

Omnigenity Optimization Through Omnigenous Phase Space Definition

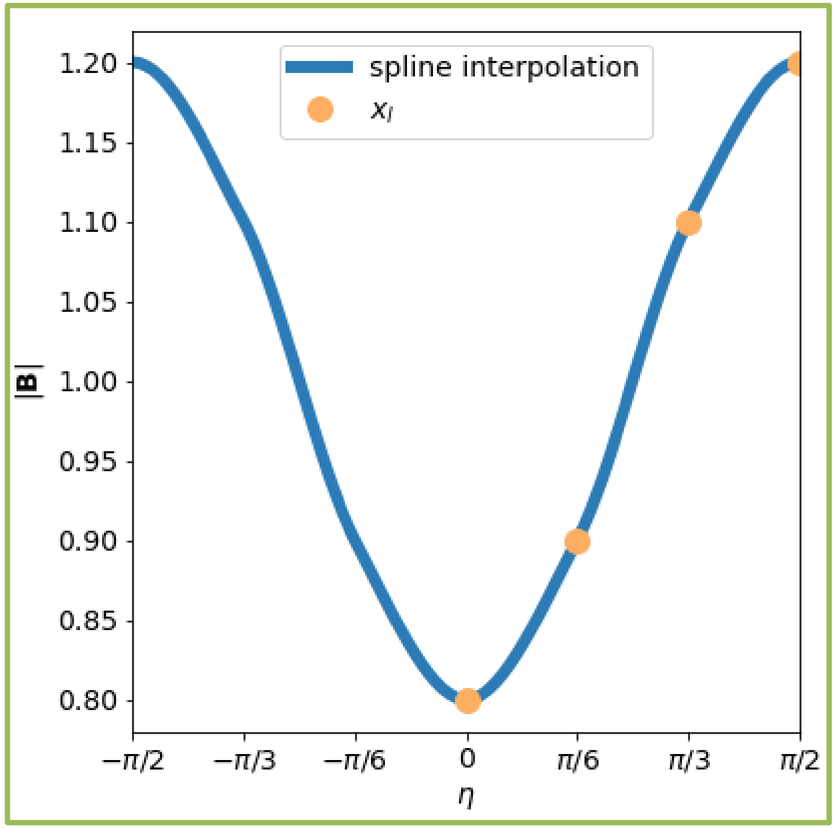
- **Idea: Create a model that can describe any omnigenous field of interest**

- **Then, in optimization we can penalize the difference between the target omnigenous field and the equilibrium's field**

$$\boldsymbol{f_{OM}} = B_{eq}(\alpha,\eta) - B_{OM}(\alpha,\eta)$$

- **Because we parametrize the omnigenous field, we are free to:**
  - **Keep parametrization the same (target a specific omnigenous field)**
  - **Allow the parametrization to be part of the optimization (optimize for SOME omnigenous field)**

Example: Full QI Phase Space Definition in DESC



- **Specify the magnetic well "shape" in computational coordinate η**

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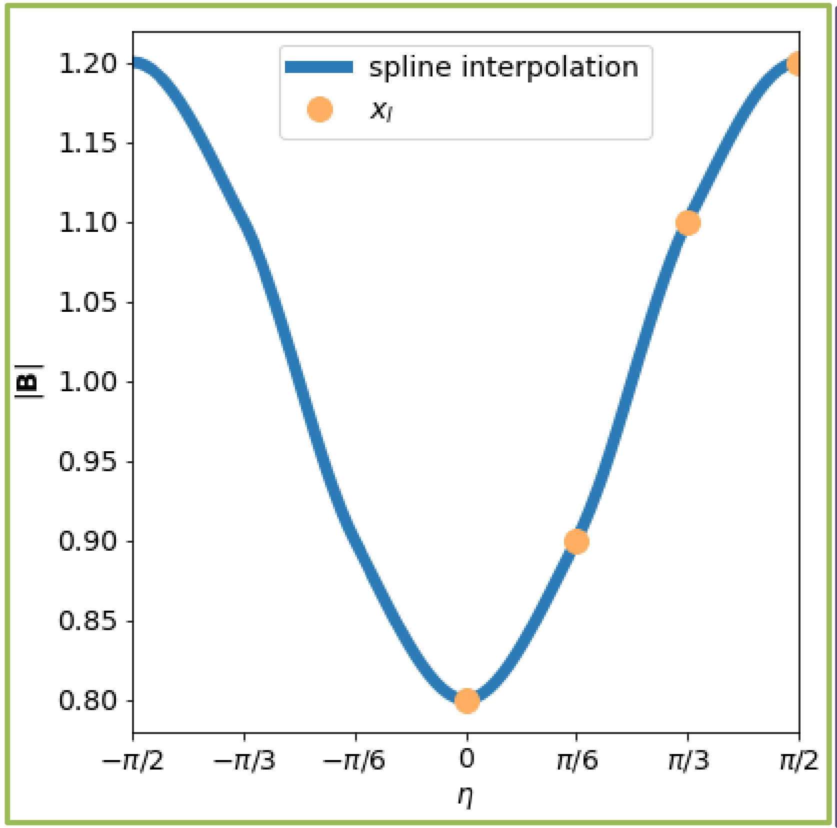
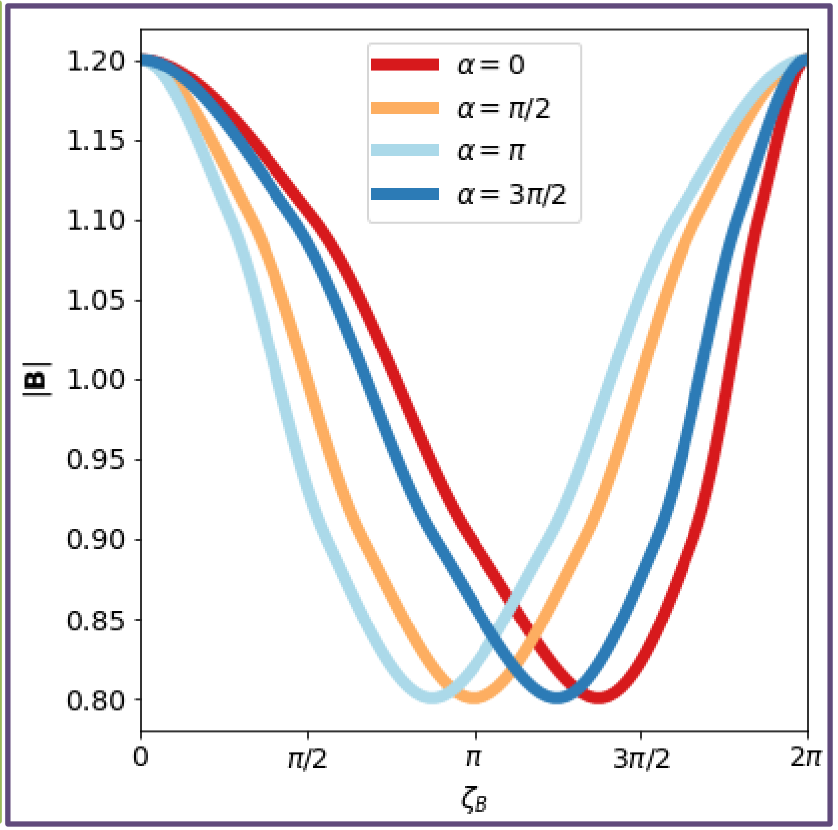
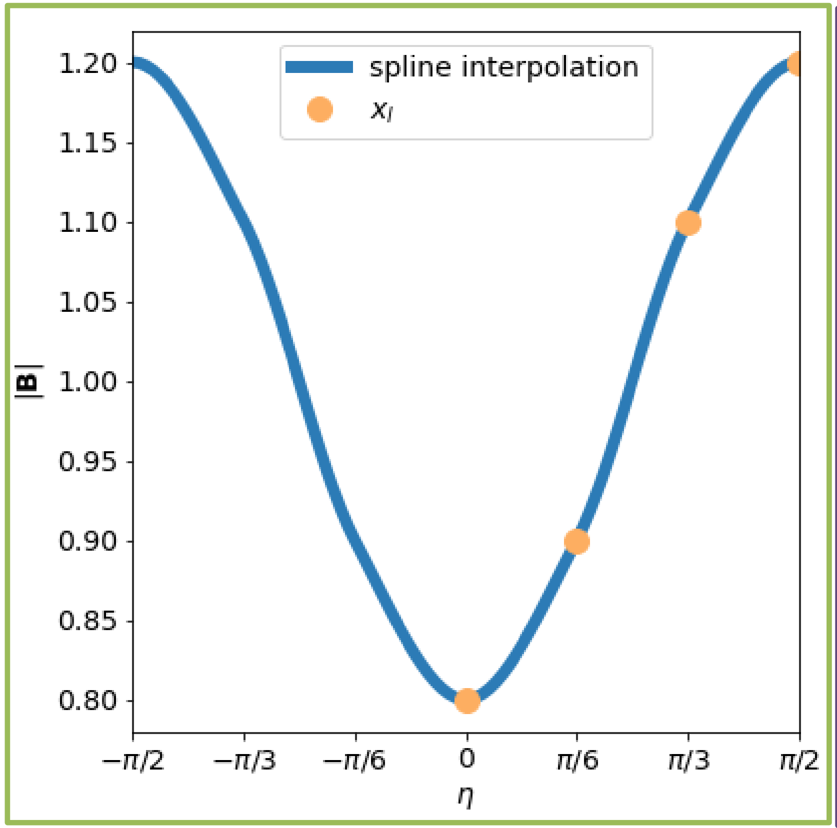
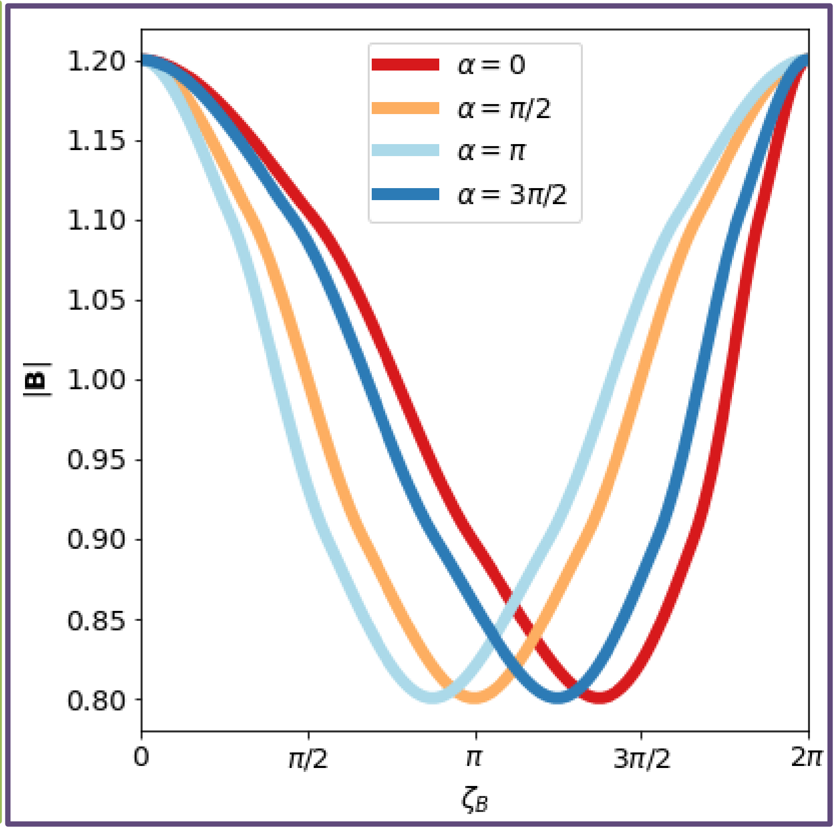


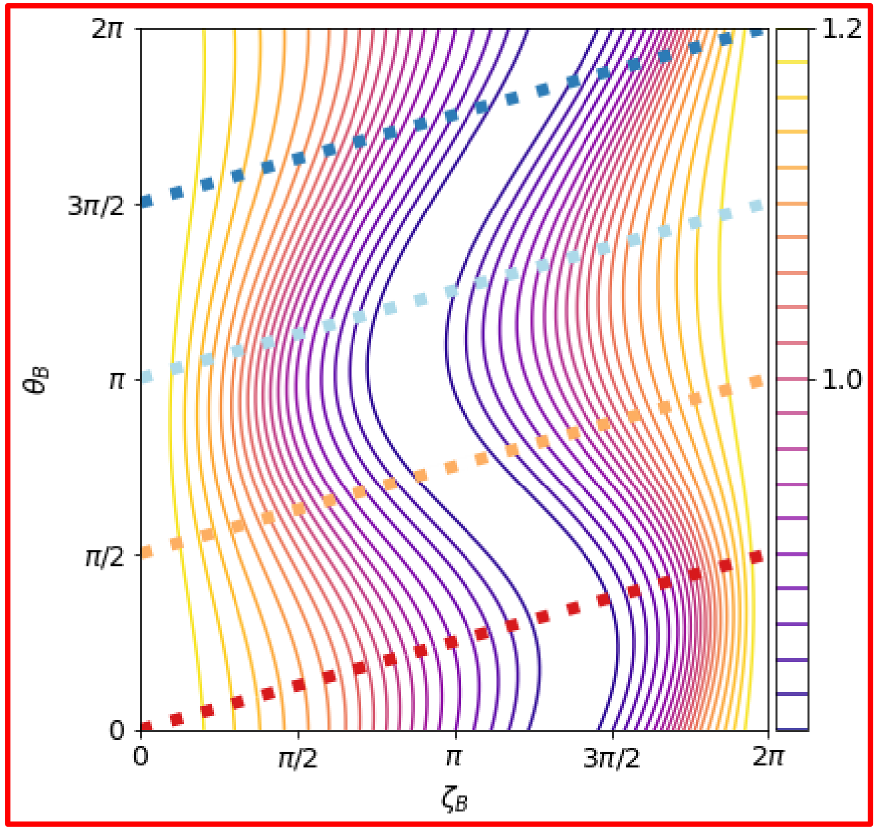


- **Specify the magnetic well "shape" in computational coordinate η**
- **Specify how the well "shifts" on different field lines with a Fourier series x_{mn} in (η, α)**

Example: Full QI Phase Space Definition in DESC



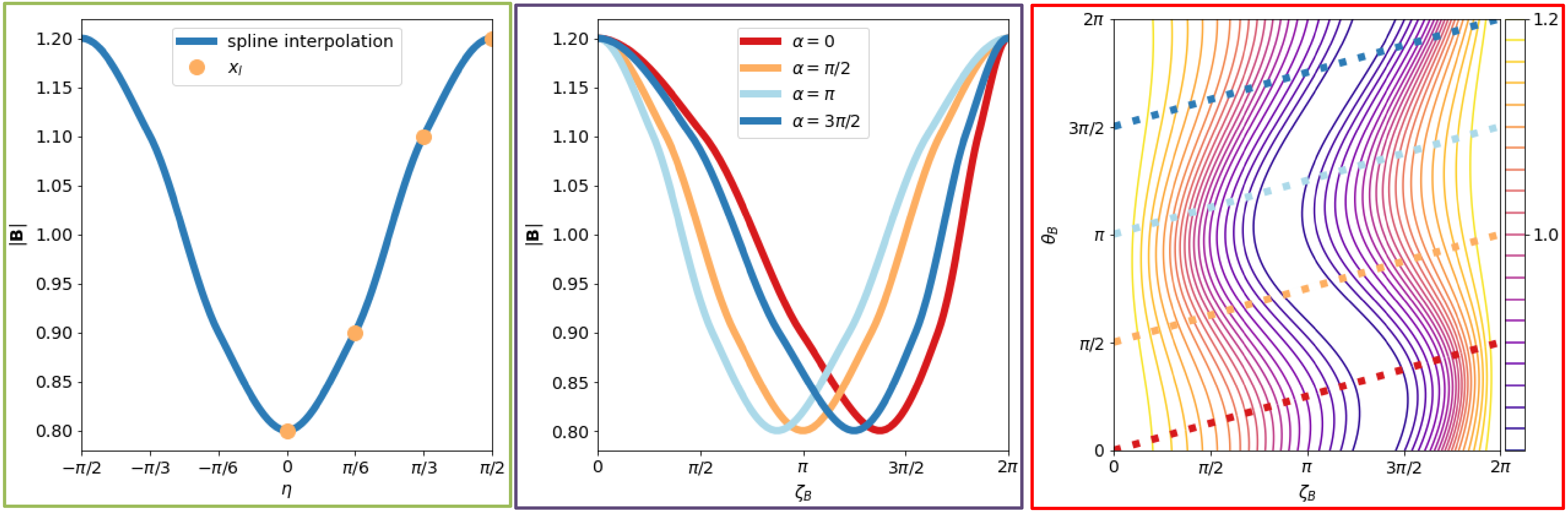




- Specify the magnetic well "shape" in computational coordinate η
- Specify how the well "shifts" on different field lines with a Fourier series x_{mn} in (η, α)
- Generate arbitrary QI magnetic field targets without prior initialization

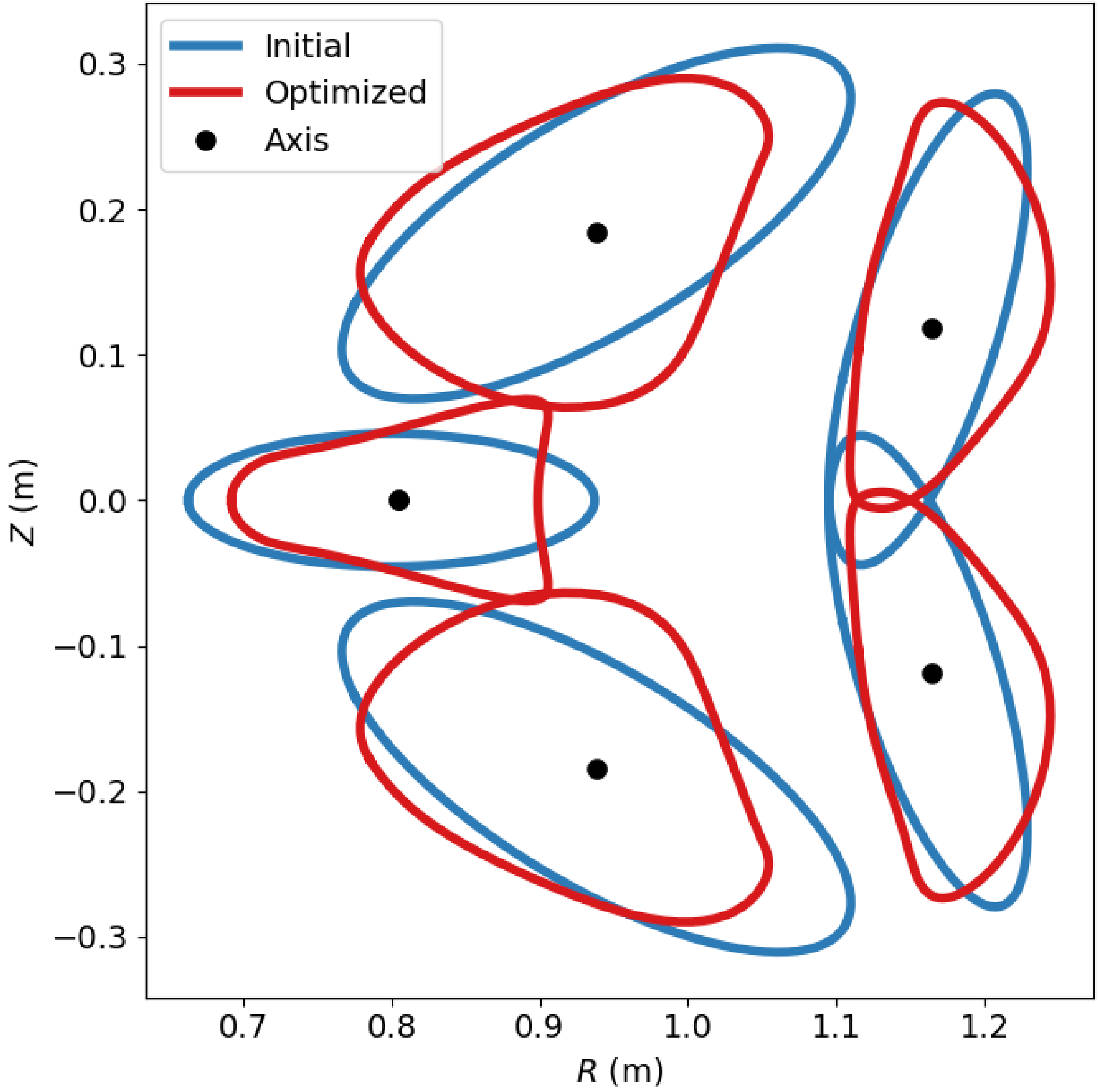
Plasma Control

Example: Full QI Phase Space Definition in DESC



- Specify the magnetic well "shape" in computational coordinate η
- Specify how the well "shifts" on different field lines with a Fourier series x_{mn} in (η, α)
- Generate arbitrary QI magnetic field targets without prior initialization
- Model enables scans of the QI optimization landscape

Example 1: unconstrained QI target



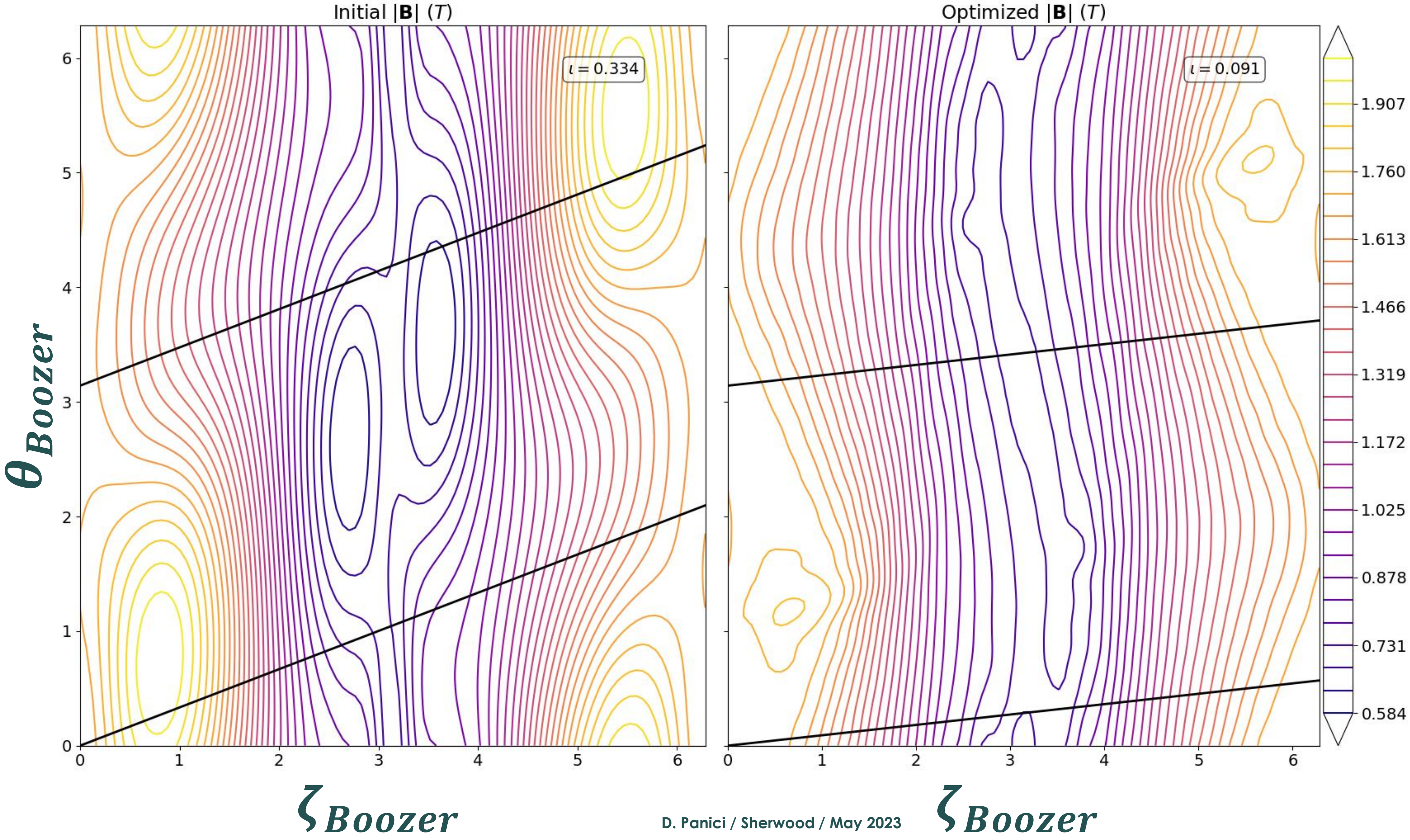
<u>Initial equilibrium:</u>

- Analytic QI model
- Fixing axis of equilibrium

<u>Optimization targets:</u>

- Unconstrained QI on multiple surfaces **(target field allowed to vary)**
- Vacuum force balance

Example 1: unconstrained QI target



D. Panici / Sherwood / May 2023

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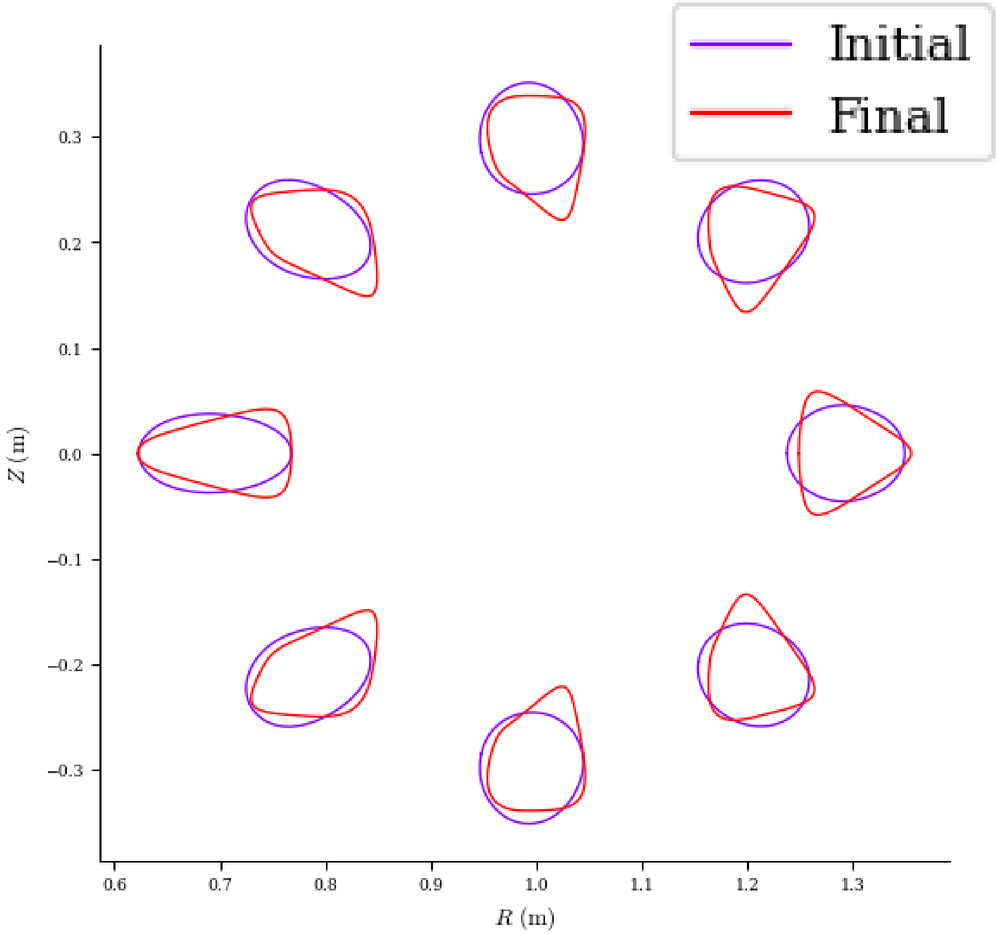
Example 2: Toroidally closed omnigenity



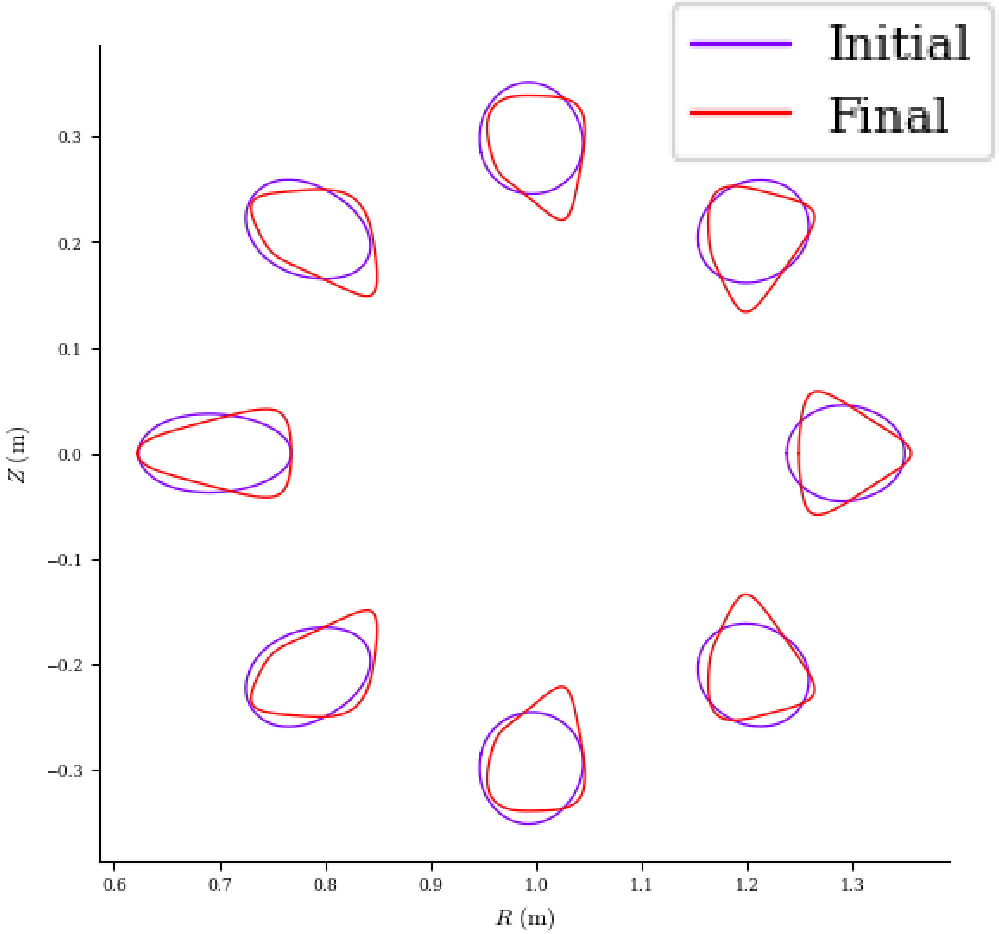
Initial equilibrium:

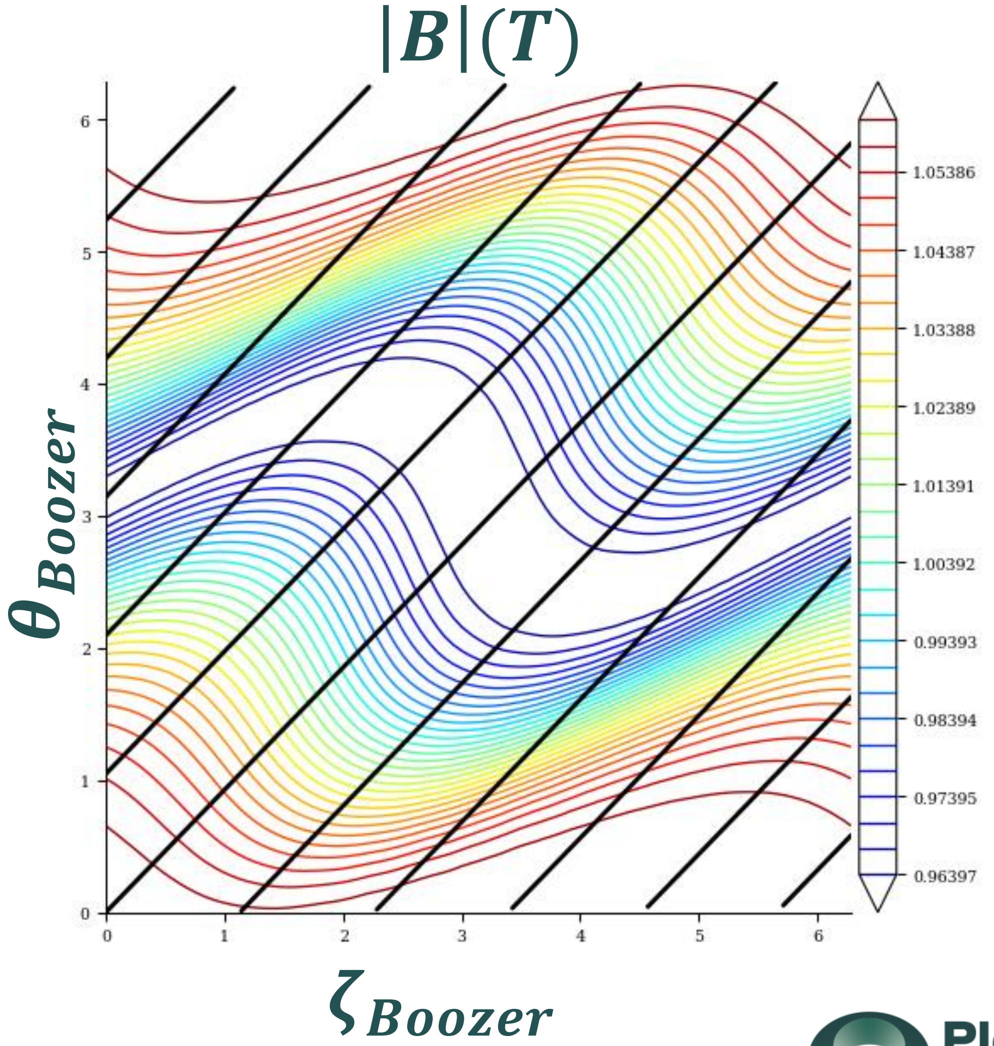
- QA NAE equilibrium from pyQSC
- Fixing axis and O(ρ) of equilibrium

Optimization targets:

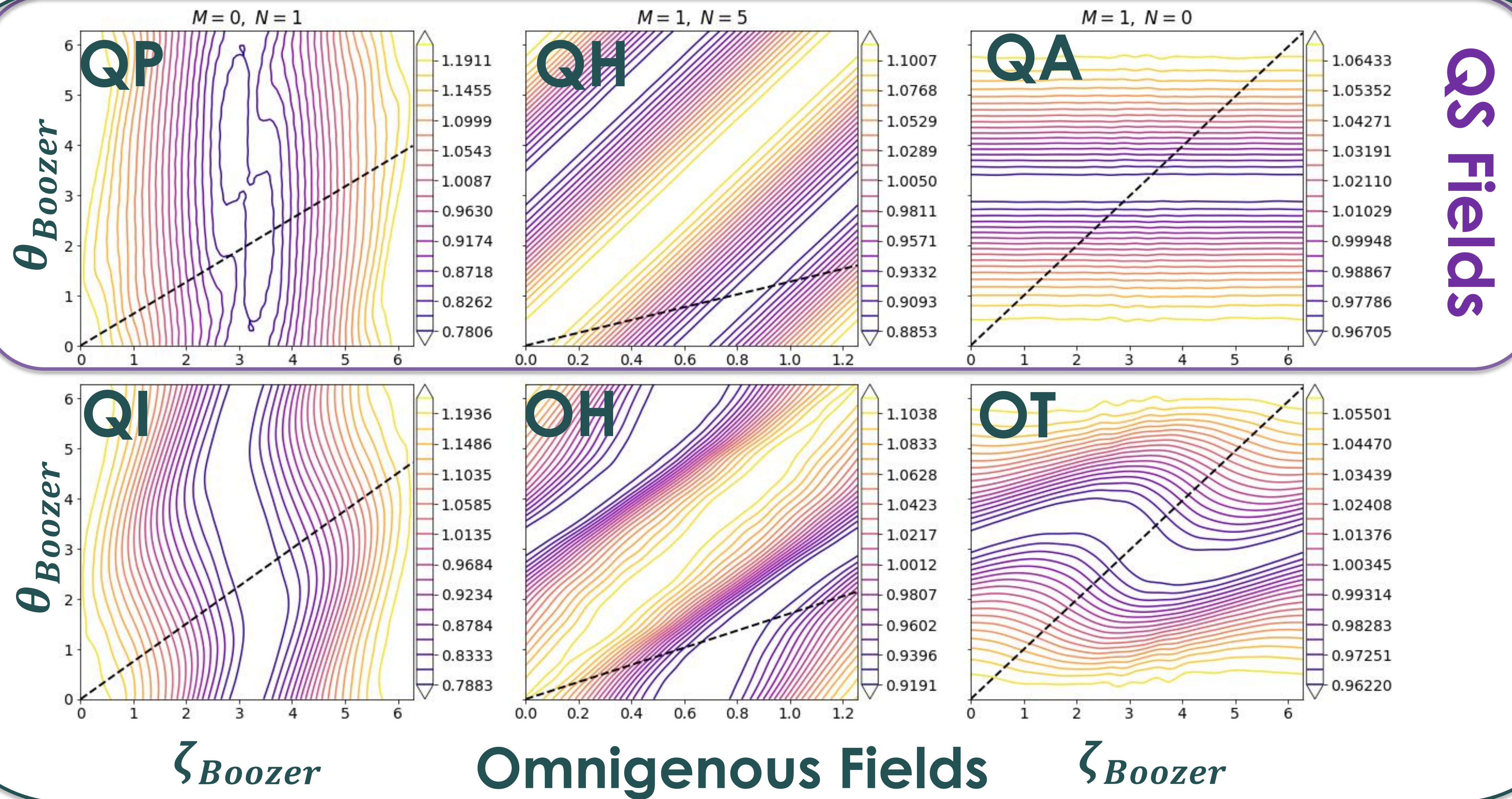
- Targeting specific omnigenous field with toroidally closed |B| contours on multiple surfaces
- Force balance at finite beta

Example 2: Toroidally closed omnigenity





DESC can find equilibria with any omnigenity type



Constrained Optimization

Stellarator optimization is full of constraints

- **Geometry / Engineering:**
  - **Aspect Ratio = 6**
  - **Minimum coil-plasma distance > 1.2m**
  - **Maximum coil curvature < 0.6**
- **Stability:**
  - **Equilibrium**
  - **Magnetic Well > 0**

- **Physics:**
  - **$Iota_{edge}$ = 5/5**
  - **J_{boot} = 0**
- **Self Consistency:**
  - **$J_{MHD} = J_{kinetic}$**
  - **$p_{MHD} = p_{kinetic}$**
  - **$B_{n,plasma} = -B_{n,coils}$**

$$\min_x f(x)$$

$$\text{subject to} \quad g_{eq}(x) = 0 \quad \text{(Equality Constraints)}$$

$$g_{ineq}(x) \geq 0 \quad \text{(Inequality Constraints)}$$

Equality Constraints

Inequality Constraints

Plasma Control

Current methods : Sum of Squares + max

$$\min_x f(x) \text{ subject to } \begin{array}{r} g_{eq}(x) = 0 \\ g_{ineq}(x) \geq 0 \end{array}$$

Combine all constraints into a single objective with different weights:

$$\min_x f(x) + w_1 [g_{eq}(x)]^2 + w_2 \max(0, g_{ineq}(x)]))^2$$

Limitations:

- **Hard to guess a-priori what weights should be**
- **Constraints are only satisfied as w → infinity**
- **Leads to badly scaled problem**
- **Non-smooth due to max term**

Better methods: Augmented Lagrangian

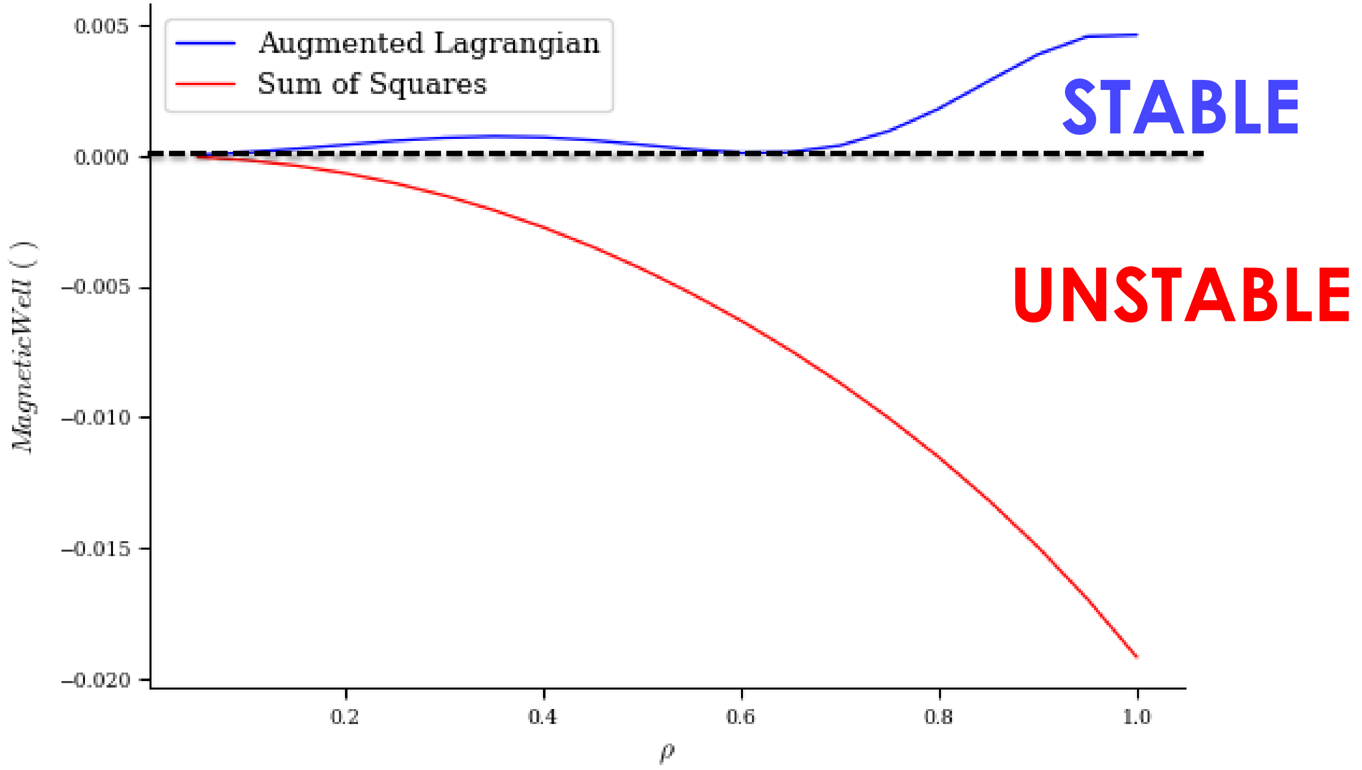
Combination of traditional Lagrangian + quadratic penalty

$$\mathcal{L}(x, \lambda, \mu) = f(x) + \lambda^T \mathbf{g}(x) + \mu g^2(x)$$

- **Smooth function**
- **Systematic way exists to increase penalty -> remove guesswork of weights!**
- **Python/JAX version implemented in DESC**

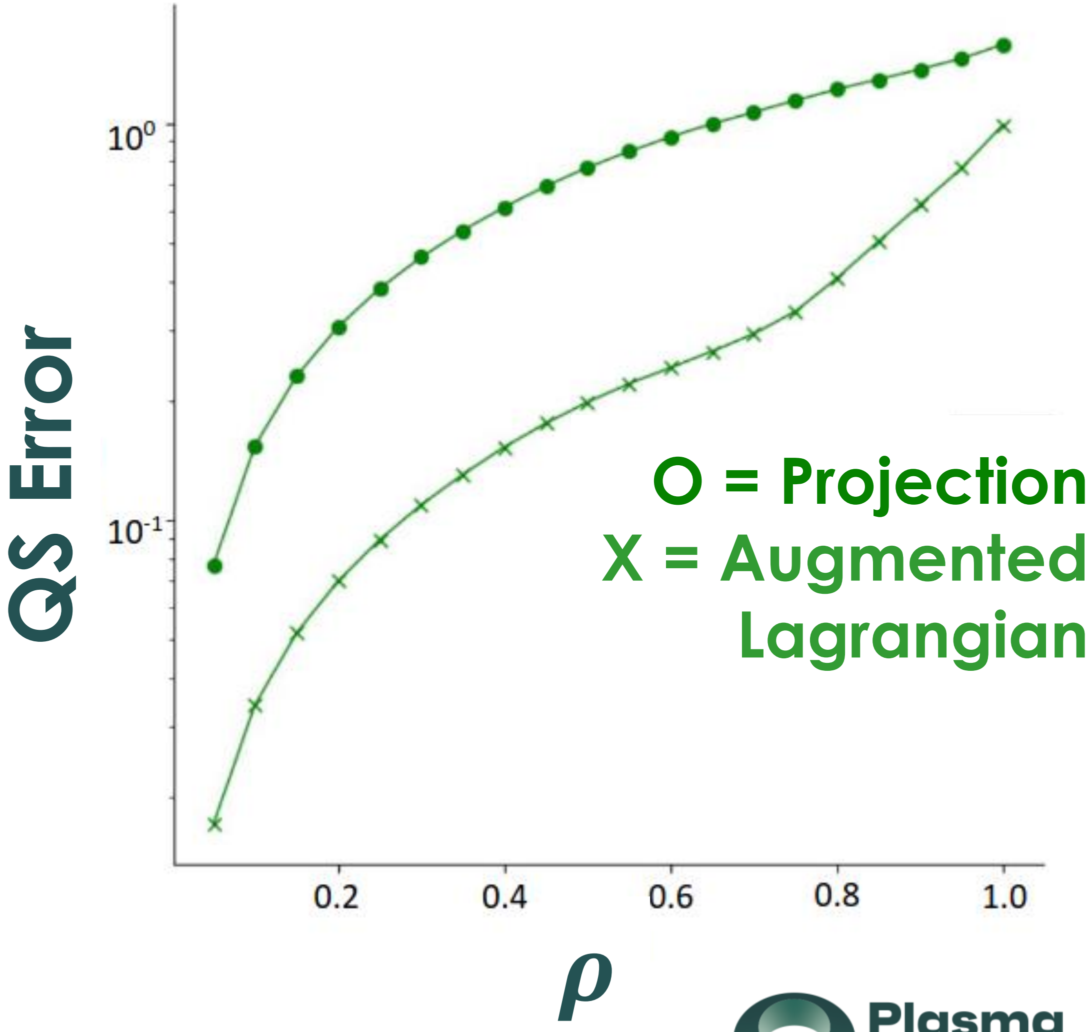
Augmented Lagrangian takes guesswork out of penalty terms

- **Simple quadratic penalty fails to give stable equilibrium, even for large values of weight**
- **Instead applying inequality constraint w/ augmented Langrangian gives magnetic well > 0**



Augmented Lagrangian approach allows better solutions to be found

- **Conventional projection method re-solves from boundary at each step, to enforce force balance**
- **Augmented Lagrangian systematically varies weighting of constraints vs objective to improve QS**
  - **Allows it to achieve better final result, without need for guesswork**

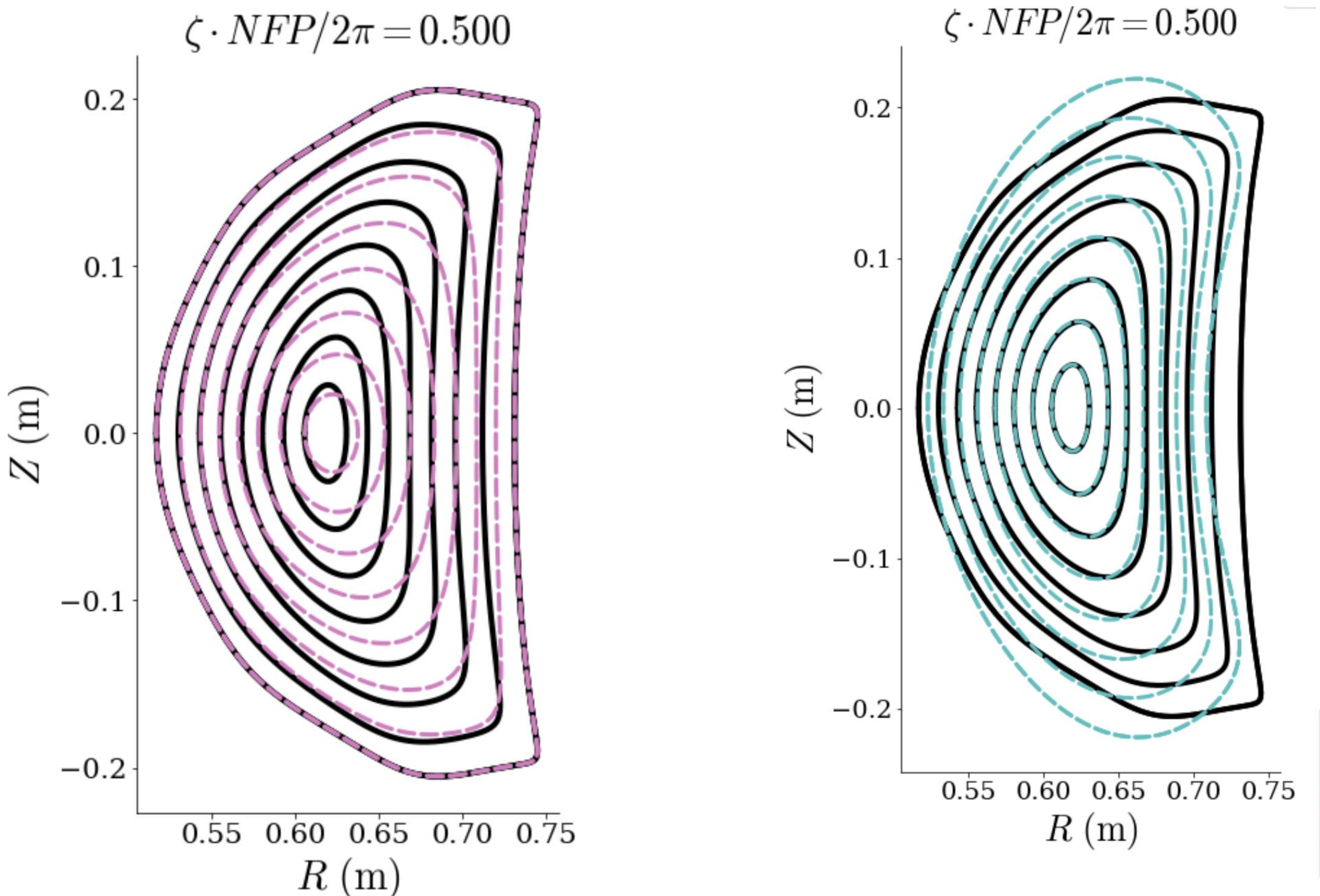


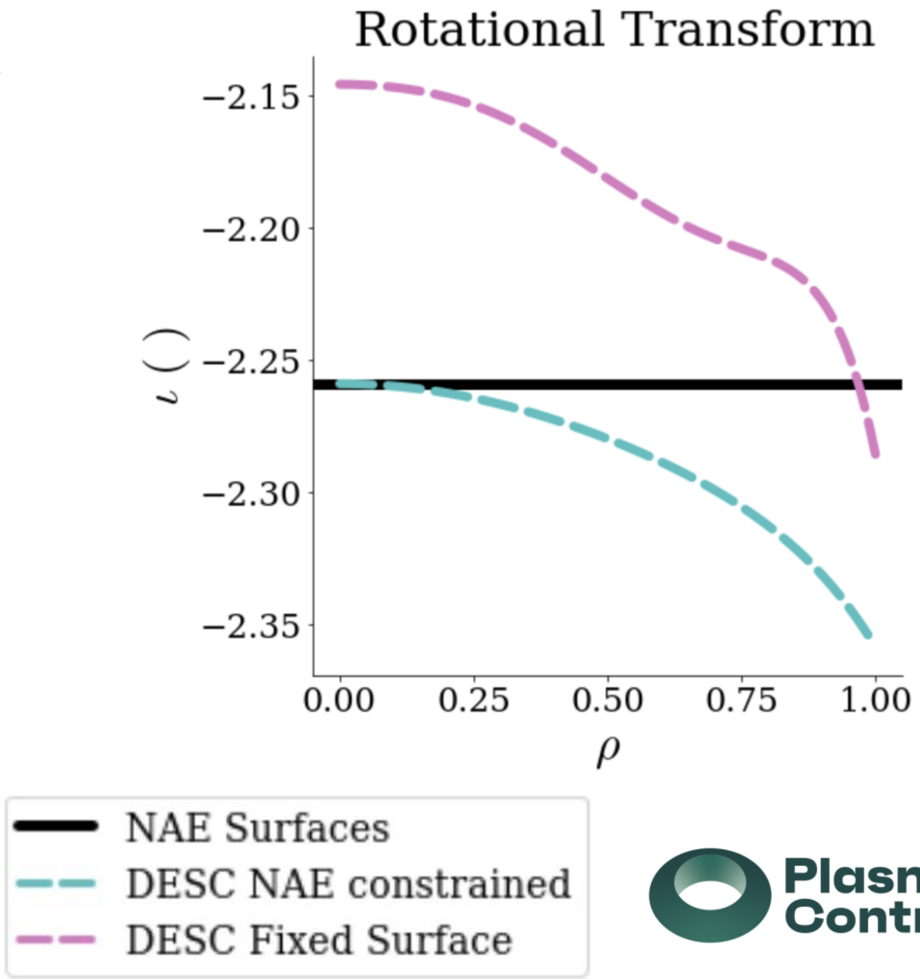
Plasma Control

Summary

DESC Offers Unique Approaches to Stellarator Optimization

Poincare and NAE-Constrained Equilibria offer **new ways to explore phase space of nested surface stellarators**





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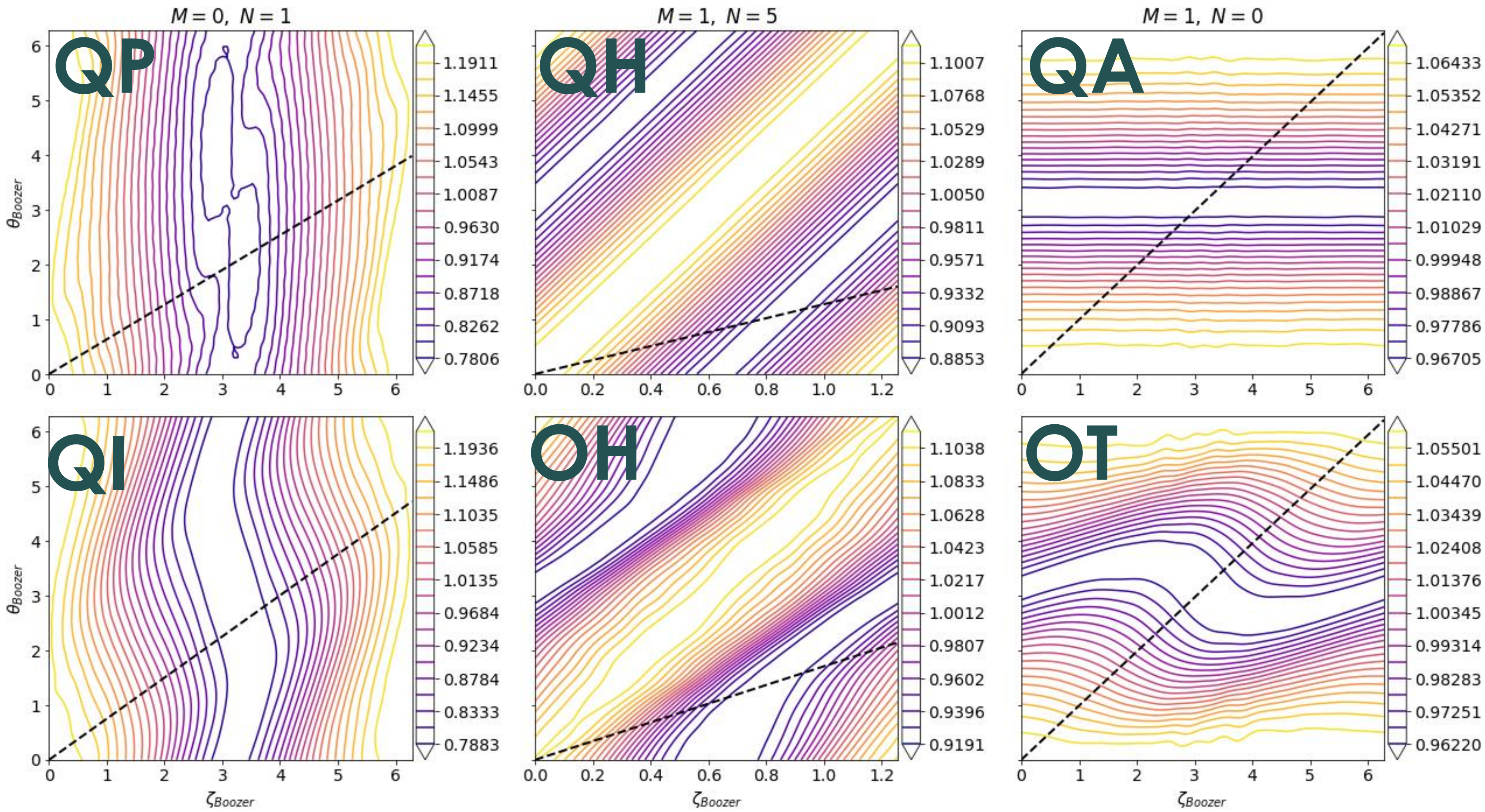
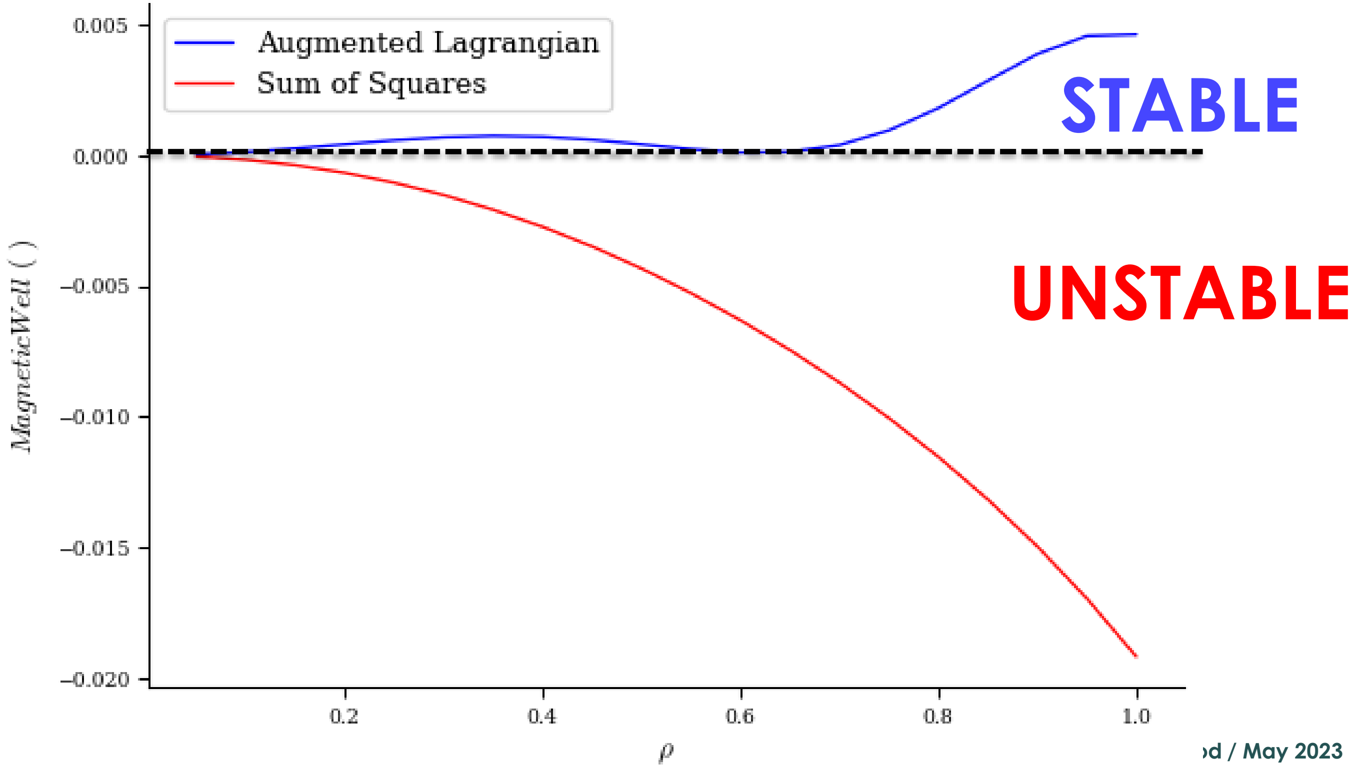
DESC Offers Unique Approaches to Stellarator Optimization

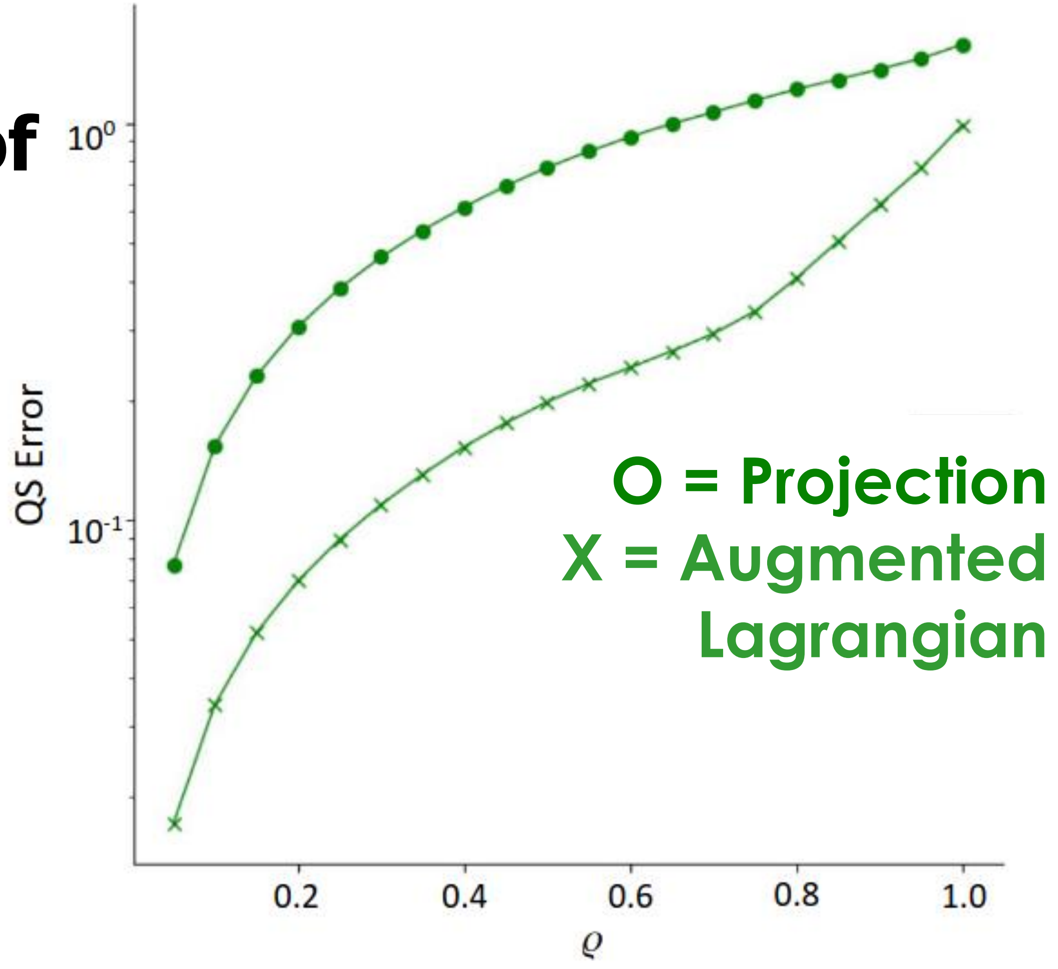
Omnigenous field phase space model allows **systematic exploration of the space of omnigenous stellarators**



DESC Offers Unique Approaches to Stellarator Optimization

- Proper Constrained Optimization in DESC can **obtain better solutions and eliminates the need for guesswork of penalty terms**





Additional Resources

Princeton Plasma Control
control.princeton.edu

Software

- **Open-source repository:** `https://github.com/PlasmaControl/DESC`
- **Python package:** `pip install desc-opt`

Papers

- **The DESC Stellarator Code Suite Part I** https://arxiv.org/abs/2203.17173
- **The DESC Stellarator Code Suite Part II** https://arxiv.org/abs/2203.15927
- **The DESC Stellarator Code Suite Part III** https://arxiv.org/abs/2204.00078

The Plasma Control group is recruiting graduate students and post-docs!
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